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# Study on Fiber Motion in Turbulent Flow

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# STUDY ON FIBER MOTION IN TURBULENT FLOW

*A*

*Thesis Submitted to the Department of Applied Mathematics,*

*Rajshahi University, Rajshahi-6205, Bangladesh*

*for the Fulfillment of the Requirements of the Degree of*

*Master of Philosophy*

*in*

*Applied Mathematics*

*BY*

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*This Work is Dedicated to*
*My Parents Whose Love*
*& Affection*
*Always Encouraged Me*

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# *CERTIFICATE*

I certify that the thesis entitled "Study on Fiber Motion in Turbulent Flow" submitted by Shams Forruque Ahmed in fulfill requirement for the degree of Master of Philosophy in Applied Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh has been completed under my supervision. I believe that this research work is an original one and it has not been submitted elsewhere for any degree.

I wish him every success in life.

(Professor Dr. M. Shamsul Alam sarker)
Supervisor

# PREFACE

The thesis entitled "Study on Motion of Fibers in Turbulent Flow" is being presented in partial fulfillment of the Requirements for the degree of Master of Philosophy.

This thesis is mainly divided into six chapters. The first one is an introductory chapter. Fundamentals of turbulence and concepts of fiber have been discussed here. Some results and theories, which are needed in the subsequent chapters, have been included in this chapter. A brief review of the past researches related to this thesis has also been given. The numbers inside brackets [] refer to the references, which are expressed alphabetically at the end of the study. In the second chapter some equations of turbulent motion has been derived which are applicable in the next chapters.

In the third chapter, the equation of motion for turbulent flow of fiber suspensions has been derived in terms of correlation tensors of second order. Mathematical modeling of fiber suspensions in the turbulent flow is discussed including the correlation between the pressure fluctuations and velocity fluctuations at two points of the flow field, where the correlation tensors are the functions of space coordinates, distance between two points and the time.

In chapter four, the equation of motion for turbulent flow has been derived in terms of correlation tensors of second order in a rotating system. Due to rotation the coriolis force plays an important role in the rotating system of turbulent flow. The results are obtained by taking correlation between the pressure fluctuations and velocity fluctuations at two points of the flow field, where the correlation tensors are the functions of space coordinates, distance between two points and the time.

In the fifth chapter, the equation of fiber motion in dusty fluid turbulent flow has been derived in terms of correlation tensors of second order. In presence of dust particles, mathematical modeling of fiber suspensions in the turbulent flow is discussed including the correlation between the pressure fluctuations and velocity fluctuations at two points of the flow field, where the correlation tensors are the functions of space coordinates, distance between two points and the time.

Finally, in the last chapter, the equation of fiber motion for dusty fluid turbulent flow has been derived in a rotating system in terms of correlation tensors of second order, where the correlation tensors are the functions of space coordinates, distance between two points and the time. The system includes the effect of coriolis force due to rotation in the fluid flow with the correlation between pressure fluctuations and velocity fluctuations at two points of the flow field.

The following research papers which are extracted from this thesis have been accepted and communicated for publication in the different reputed Journals:

1. Motion of Fibers in Turbulent Flow in a Rotating System (Accepted for publication in the Journal, Rajshahi University Journal of Science).
2. Fiber suspensions in Turbulent Flow with Two-Point Correlation (Presented in the $5^{th}$ Asian Mathematical Conference held on Putra World Trade center, Kuala Lumpur, Malaysia and this paper is considered for publication in the proceedings).

3.Fiber Motion in Dusty Fluid Turbulent Flow with Two-Point Correlation (Communicated for publication).

4.Fiber Motion in Dusty Fluid Turbulent Flow in a Rotating System (Communicated for publication).

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## General Introduction

### 1.1. Introductory to Turbulent Flow

Turbulence means agitation, commotion and disturbance. This definition is, however too general and does not suffice to characterize turbulent fluid motion in the modern sense. Osborn Reynolds in the study of turbulent flows, named this type of motion "sinuous motion". The use of the word "turbulent" is to characterize a certain type of flow, namely the counterpart of streamline motion. In fluid dynamics, turbulence or turbulent flow is a fluid regime characterized by chaotic, stochastic property changes. This includes low momentum diffusion, high momentum convection, and rapid variation of pressure and velocity in space and time.

Turbulence occurs nearly everywhere in nature. It is characterized by the efficient dispersion and mixing of vorticity, heat, and contaminants. In flows over solid bodies such as airplane wings or turbine blades, or in confined flows through ducts and pipelines, turbulence is responsible for increased drag and heat transfer. Turbulence is therefore a subject of great engineering interest. On the other hand, as an example of collective interaction of many coupled degrees of freedom, it is also a subject at the forefront of classical physics.

Origin of turbulence is a central role in determining the state of fluid motion played by the Reynolds number. In general, a given flow undergoes a succession of instabilities with increasing Reynolds number and, at some point, turbulence appears more or less abruptly. It has long been thought that the origin of turbulence can be understood by sequentially examining the instabilities.

In 1937, Taylor and Von Karman [29] gave the definition,

"*Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another.*"

According to this definition, the flow has to satisfy the condition of irregularity. This irregularity is a very important feature. Because of irregularity, it is impossible to describe the motion in all details as a function of time and space coordinates. But turbulent motion is irregular in the sense that it is possible to describe it by the laws of probability. It appears possible to indicate distinct average values of various quantities, such as velocity, pressure, temperature etc. If turbulent motion were entirely irregular, it would be inaccessible to any mathematical treatment. Therefore, it is not sufficient to say that turbulence is an irregular motion.

According to J.O. Hinze [11], the turbulent flow is

"*Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with space and time coordinates, so that statistically only distinct average values can be discerned.*"

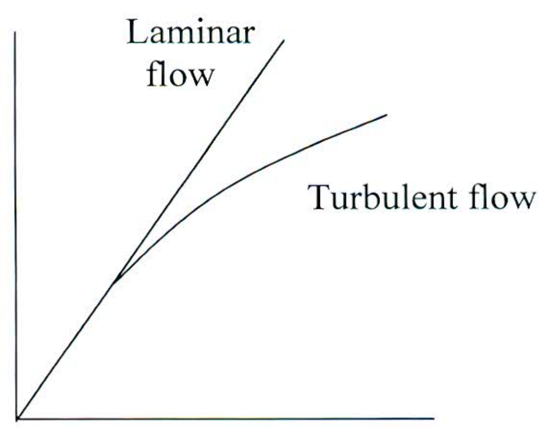
The addition "with space and time coordinates" is necessary; it is not sufficient to define turbulent motion as irregular in time alone. For instance, the case in which a given quantity of a fluid is moved bodily in an irregular way; the motion of each part of the fluid is then irregular with respect to time to a stationary observer, but not to an observer moving with the fluid. Again, turbulent motion is not irregular in space alone, because a steady flow with an irregular flow pattern might then come under the definition of turbulence.

According to the definition of Taylor and Von Karman [29] there are two distinct types of turbulence, wall turbulence and free turbulence.

***Wall Turbulence:*** Turbulence generated by a viscous effect due to presence of a solid wall is designated by wall turbulence.

***Free Turbulence:*** Turbulence in the absence of wall generated by the flow of layers of fluids at different velocities is called free turbulence.

Turbulent flow occurs in our daily life. If we observe the smoke rising out of a chimney of a factory or a cigarette, we find that upto a certain length from the chimney or the cigarette, the smoke has a regular shape and after that its shape becomes irregular and if we see still farther then the smoke becomes completely irregular. Again, if a drop of ink is dropped in a glass of water, we find a similar phenomenon, i.e, a regular ink thread falling for a short distance after which it spreads and a vortex type motion can be observed. Ultimately the thread splits into several vortices and motion becomes irregular. The flows with such irregular motions are usually called turbulent flows. Turbulent flow also occurs in large arteries at branch points, in diseased and narrowed (stenotic) arteries and across stenotic heart valves.




Effects of turbulence on the pressure-flow relationship

Osborne Reynolds [21] was first systematically investigated the transition from laminar to turbulent flow. In his experiments, Reynold used a glass tube with flowing water from a reservoir and observed the flow pattern by injecting a thin stream of dye into the main stream. We find that if the velocity of the water is small, then there is a regular thread of dye moving throughout the tube. As the velocity of the fluid is increased, the thread of dye after a short distance from the point of injection becomes irregular. This irregularity in the shape of the thread increases with the increase of the velocity. We also find that the length of the thread of the dye, which is regular decreases with the increase of velocity, indicating that irregular motion starts developing at a smaller distance if the fluid velocity is increased. We also find that if the viscosity of the fluid in the pipe or the channel is small then the irregular motion develops much quicker than when the viscosity is high. All these observations can be combined to say that the turbulent flow sets in if the Reynolds number is sufficiently high.

The origin of the idea of statistical approach of turbulence traced by Taylor [27] in which he has advanced the concept of Lagrangian correlation coefficient that provides a theoretical basis for turbulent diffusion. The most important work done by Taylor [28] is that he gives up the old theories of turbulence based on the kinetic theory of gases and introduces the idea that the velocity of the fluid in turbulent motion is a random continuous function of position and time. He introduces the concept of correlation between velocities at two points.

The flows which occur in practical applications are turbulent. The study of turbulent flows is very important both from theoretical as well as practical points of view, because most of natural phenomena connected with fluid flows involve turbulence. As for example, the flows in rivers, natural streams, the winds in the atmosphere, the motion of clouds in the rainy season, flow in water supply pipe, flow in fluid machinery such as fans, pumps, turbines etc. are turbulent flows occurring in our daily life. This flow usually differs from the streamline flow or the laminar flow of a viscous fluid and occurs at a high Reynolds number. The occurrence of the turbulent flow will depend on the values of the non-dimensional number called critical

Reynolds number and this number varies from 2,000 to 2,300. If the Reynolds number ($R_e$) is greater than the critical Reynolds number ($R_{cr}$) then the flow will be turbulent and if the Reynolds number is less than the critical Reynolds number then the flow will be laminar. Transition normally takes place at Reynolds number 2,000 to 4,000.

The transition mechanism is rather complicated and is still not fully understood. For examples, when extreme preconditions were taken in minimizing the initial disturbances in pipe flow. Transition could be delayed until a Reynold number of 50,000 has been attained. Thus the transition Reynold number seems to depend partly on the degree of turbulence in the flow, its numerical value always increases with decrease in turbulence.

Transition phenomenon has been associated with the stability of laminar flow. The stability of laminar boundary layer with zero pressure gradient was investigated by Toltman, using small perturbation. It is found that laminar boundary layer is completely stable with respect to small disturbance at the value of $R_e = \frac{U\delta^*}{\upsilon} < 575$, where $\delta^*$ is the displacement thickness. Since, $\delta^* = 1.73\sqrt{\frac{\upsilon x}{U}}$, it implies that the laminar flow is stable for the value of $R_e = \frac{Ux}{\upsilon} < 1.1\times10^5$. Hence Reynolds number below $1.1\times10^5$, disturbances are damped and flows retain its laminar form. At high Reynolds number disturbance may be amplified and the transition process is only initiated by amplification of the disturbance, it follows that transition must occurs downstream of the point for which Reynolds number is $1.1\times10^5$.

The stability of laminar flow is greatly affecting transition from laminar flow to a turbulent flow in presence of pressure gradient. For accelerated flow

($\frac{dP}{dx}<0, \frac{dU}{dx}>0$), the critical Reynolds number increases, i.e, increases in stability where as far related flow ($\frac{dP}{dx}>0, \frac{dU}{dx}<0$), critical Reynolds number decreases, i.e, transition to turbulent flow much more easily provoked.

***Mean Motion and Fluctuations:***

To describe a turbulent motion quantitatively it is necessary to introduce the notion of scale of turbulence; a certain scale in time and space. It is not sufficient to characterize a turbulent motion by its scale alone.

Since, in a turbulent flow the dependent variables have random behavior besides some mean behavior; we can only describe the flow in terms of some average quantities. In the analysis we usually take two-types of averages; one is the average with respect to time and the second is average with respect to space.

In mathematical description of turbulent flow, it is convenient to consider an instantaneous velocity, such as $u$ is the sum of the time-average part $\bar{u}$ and momentary fluctuation part $u'$, i.e, $u = \bar{u} + u'$,

where, the quantity with bar denotes the mean value or average value and the quantity with prime denotes the fluctuating value. The mean value and the fluctuating value can be shown in the following figure:

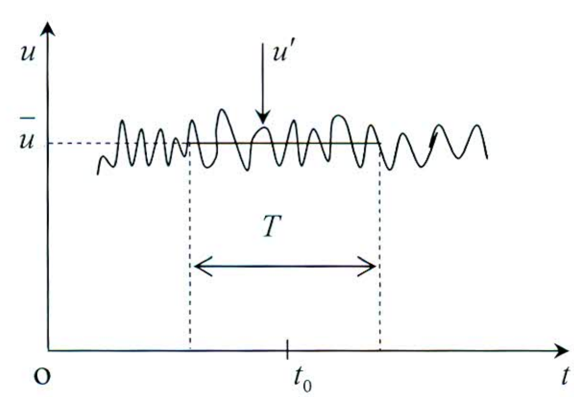



**Figure-1.1: Time averaging for a statistically steady flow**

In a steady flow $\bar{u}$ does not change with time. Average values can be determined in various ways. If the turbulent flow field is quasi-steady or stationary random, averaging with respect to time can be used. In the case of homogeneous turbulence flow field, averaging with respect to space can be considered. It is not always possible to take time-mean and space-mean values if the flow field is neither steady nor homogeneous. In such a case we may assume that an average [19] is taken over a large number of experiments that have the same initial and boundary conditions. We then speak of an ensemble-mean value.

An averaging procedure can be carried out only if certain conditions are satisfied. There are various methods of averaging may be expressed in mathematical form. If we use the Eulerian description of the flow field, one of the three methods of averaging may have to be applied to a varying quantity at any point in the flow field.

The methods of averaging are:

i) Time average for a stationary turbulence at a point of the flow field

$$[u(x,t)]_t = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u(x,s)ds$$

In practice the scale used in averaging process determines the value of the period $2T$.

ii) Space average for homogeneous turbulence in which we take the average over all the space at a given time

$$[u(x,t)]_s = \lim_{V_b\to\infty} \frac{1}{V_b} \int_{V_b} u(s,t)ds$$

In practice the volume scale used in averaging process determines the volume of the space $V_b$.

iii) Space-time average in which we take the average over a long period of time and over the space and is defined as

$$[u(x,t)]_{s,t} = \lim_{T\to\infty, V_b\to\infty} \frac{1}{2TV_b} \int_{-T}^{T} \int_{V_b} u(s,y)\,ds\,dy$$

In practice the values of $T$ and $V_b$ are determined by the scale used.

iv) Statistical average in which we take the average over the whole collection of sample turbulent fluctuations for a fixed point in space and at a fixed time is defined as

$$[u(x,t,w)]_w = \int_{\Omega} u(x,t,w)\,d\mu(w)$$

over the whole $\Omega$ space of $w$, the random parameter. The measure is

$$\int_{\Omega} d\mu(w) = 1$$

A random scalar function $u(x,t,w)$ is a function of the spatial coordinates $x$ and time $t$. The parameter $w$ is chosen at random according to some probability law in the space $\Omega$. If the flow is unsteady, time average cannot be used and must be replaced by ensemble averaging and the ensemble average of $N$ identical experiments are defined as

$$[u_n(x,t)]_e = \lim_{N\to\infty} \sum_{n=1}^{N} \frac{u_n(x,t)}{N}$$

where, $N$ is the numbers of members of ensemble which must be large enough to eliminate the effect of fluctuations. This type of averaging can be applied to any kind of flow. For stationary homogeneous turbulence we may except and assumed that the three averaging lead to same result

$$[u(x,t)]_s = [u(x,t)]_t = [u(x,t)]_e$$

This assumption is known as the ergodic hypothesis.

For an unsteady flow, the ensemble averaging can be shown by the following figure:

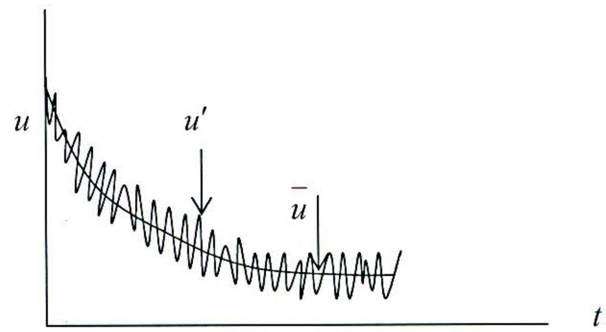



**Figure-1.2: Ensemble averaging for unsteady flow**

Actual turbulent flows are neither really stationary nor homogeneous. Therefore for practical reason, we cannot carry out the averaging procedure with respect to time or space for infinite values of $T$ and $X$ respectively but only for a finite values. For instance, consider averaging with respect to time of the Eulerian velocity of turbulent flow. The flow may contain very slow variation that we do not wish to regard as belonging to the turbulent motion of the flow. In taking average with respect to time, we have to take some suitable time scale. Let $q$ be the velocity at any point. Then we define the time average velocity or simply the mean velocity at a time $t_0$ as

$$\bar{q} = \frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} q\,dt$$

where, $T$ is the time scale of the average.

The time interval $T$ must be taken sufficiently large compared with the time scale of turbulent fluctuations, but small compared with time scale of any slow changes in the flow field, which are not associated with turbulence.

Thus, the average value of $q$ will depend upon $T$. For $T \to \infty$, we define the time average velocity or simply the mean velocity at time $t_0$ as

$$\bar{q} = \lim_{T \to \infty} \frac{1}{T} \int_{t_0 - \frac{T}{2}}^{t_0 + \frac{T}{2}} q\,dt$$

Similarly, we can define the time average of the variables as density, pressure, temperature and also the space average of the variables.

## 1.2. Reynolds Rules of Averaging

The process of averaging [21] is that the velocity consists of two parts; one is the mean velocity $\bar{q}$ and the other is the fluctuating velocity $q'$. The same notation is used for other variables. Suppose, $q(u, v, w), p$ and $\rho$ be the instantaneous velocity, pressure, and density respectively. Reynolds was introduced elementary statistical motion into the consideration of turbulent flow. In the theoretical investigations of turbulence, he assumed the physical quantities in the flow field as

$$u = \bar{u} + u', \quad v = \bar{v} + v', \quad \rho = \bar{\rho} + \rho', \quad p = \bar{p} + p'$$

In the study of turbulence we have to carry out an averaging procedure not only on single quantities but also on product of quantities. Consider

three arbitrary statistically dependent physical quantities $A, B$ and $C$, each containing two parts due to mean motion and fluctuation. Thus, these quantities can be expressed as

$$A = \bar{A} + A', B = \bar{B} + B', C = \bar{C} + C'$$

where, $\bar{A}, \bar{B}, \bar{C}$ are the average or mean part and $A', B', C'$ are the fluctuating part of the quantity $A, B, C$ respectively and the mean of the fluctuating part should be zero. i.e, $\overline{A'} = 0, \overline{B'} = 0, \overline{C'} = 0$

Now, the mean value of the product of any two quantities out of three, i.e,

$$\overline{AB} = \overline{\left(\overline{A} + A'\right)\left(\overline{B} + B'\right)}$$

$$= \overline{\overline{A}\,\overline{B} + \overline{A}B' + A'\overline{B} + A'B'}$$

$$= \overline{\overline{A}\,\overline{B}} + \overline{\overline{A}B'} + \overline{A'\overline{B}} + \overline{A'B'}$$

$$= \overline{\overline{A}}\,\overline{\overline{B}} + \overline{\overline{A}}\,\overline{B'} + \overline{A'}\,\overline{\overline{B}} + \overline{A'}\,\overline{B'}$$

$$= \overline{\overline{A}}\,\overline{\overline{B}}$$

Consequently, $\overline{\overline{A}\,\overline{B}} = \overline{\overline{A}}\,\overline{\overline{B}} = \overline{A}\,\overline{B}$

It is noted that the average of a product is not equal to the product of the averages. The terms such as $\overline{A'B'}$ are called correlations. Average of the product of these three quantities is

$$\overline{ABC} = \overline{\left(\overline{A} + A'\right)\left(\overline{B} + B'\right)\left(\overline{C} + C'\right)}$$

or, $$\overline{ABC} = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B'C'} + \overline{B}\,\overline{A'C'} + \overline{C}\,\overline{A'B'} + \overline{A'B'C'}$$

Also it can be shown that

$$\overline{\frac{\partial A}{\partial S}} = \frac{\partial \overline{A}}{\partial S}$$

and $$\overline{\int A dS} = \int \overline{A} dS$$

***Mathematical Analysis:***

The time average of any fluctuating quantity $u'$ is

$$\overline{u'} = \lim_{T\to\infty} \frac{1}{T} \int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} u' dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \left(u - \overline{u}\right) dt$$

or, $$\overline{u'} = \lim_{T\to\infty} \frac{1}{T} \int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} u dt - \lim_{T\to\infty} \frac{1}{T} \int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \overline{u} dt$$

or, $$\overline{u'} = \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} u\,dt - \lim_{T\to\infty}\frac{\bar{u}}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} dt$$

$$= \bar{u} - \frac{\bar{u}}{T}\left[t\right]_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}}$$

$$= \bar{u} - \frac{\bar{u}}{T}.T$$

$$= \bar{u} - \bar{u} = 0$$

i.e, $\overline{u'} = 0$

Similarly, $\overline{v'} = \overline{w'} = \overline{p'} = \overline{\rho'} = 0$

Although the time average of the fluctuating quantities are zero, the quantities $\overline{u'^2}, \overline{u'v'}, \overline{u'w'}$ etc. are the time average of the product of any two fluctuation components will not necessary equal to zero.

Now, we calculate the time average of the quantities $uv$ is

$$\overline{uv} = \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} uv\,dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \left(\bar{u}+u'\right)\left(\bar{v}+v'\right)dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \left(\bar{u}\,\bar{v}+\bar{u}v'+u'\bar{v}+u'v'\right)dt$$

or, $$\overline{uv} = \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \overline{u}\overline{v}\,dt + \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \bar{u}v'\,dt$$

$$+ \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} u'\bar{v}\,dt + \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} u'v'\,dt$$

or, $$\overline{uv} = \lim_{T\to\infty}\frac{\overline{u}\overline{v}}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} dt + \lim_{T\to\infty}\frac{\bar{u}}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} v'\,dt$$

$$+ \lim_{T\to\infty}\frac{\bar{v}}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} u'\,dt + \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} u'v'\,dt$$

or, $$\overline{uv} = \overline{u}\overline{v} + \overline{uv'} + \overline{vu'} + \overline{u'v'}$$

Since, the time average of any fluctuating quantity is zero then $\overline{u'} = 0, \overline{v'} = 0$ so that the above equation gives

$$\overline{uv} = \overline{u}\,\overline{v} + \overline{u'v'}$$

Similarly, $\overline{vw} = \overline{v}\,\overline{w} + \overline{v'w'}$ and $\overline{uw} = \overline{u}\,\overline{w} + \overline{u'w'}$

The time average of the time derivative of a variable $v$, i.e,

$$\overline{\frac{\partial v}{\partial t}} = \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}}\frac{\partial v}{\partial t}dt = \lim_{T\to\infty}\frac{1}{T}\left[v\right]_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \tag{1.1.1}$$

Also, $$\frac{\partial \overline{v}}{\partial t} = \frac{\partial}{\partial t}\left\{\lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} v\,dt\right\}$$

or, $$\frac{\partial \overline{v}}{\partial t} = \lim_{T\to\infty}\frac{1}{T}\left\{\frac{\partial}{\partial t}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} v\,dt\right\} = \lim_{T\to\infty}\frac{1}{T}\left[v\right]_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}} \tag{1.1.2}$$

Therefore, from equation (1.1.1) and (1.1.2) we can write

$$\overline{\frac{\partial v}{\partial t}} = \frac{\partial \overline{v}}{\partial t}$$

Similarly, the time average of the space-derivative of a variable v,

$$\overline{\frac{\partial v}{\partial x}} = \lim_{T\to\infty}\frac{1}{T}\int_{t_0-\frac{T}{2}}^{t_0+\frac{T}{2}}\frac{\partial v}{\partial x}dt$$

It is useful here for several rules of operating on mean-time averages, as they will be required for reference. The feature which is of fundamental importance for the turbulent motion consists in the circumstance that the fluctuations $u', v', w'$ increase the mean motion $\overline{u}, \overline{v}, \overline{w}$ in such a way that the later exhibits an apparent increase in the resistance.

## 1.3. Homogeneous and Isotropic Turbulence

In the case of real viscous fluids, viscosity effects will result in the conversion of kinetic energy of flow into heat; thus turbulent flow, like all flow of such fluids is dissipative in nature. If there is no continuous external force of energy for the continuous generation of the turbulent motion, the motion will decay. Other effects of viscosity are to make the turbulence more homogeneous and to make it less dependent on direction. In the extreme case, there are two types of turbulence, homogeneous and isotropic.

***Homogeneous Turbulence:*** The turbulence that has quantitatively the same structure in all parts of the flow field is called homogeneous.

***Isotropic Turbulence:*** The turbulence is called isotropic, if its statistical features have no preference for any direction and minimum number of quantities and relations are required to describe its structure and behavior.

We imagine an infinite uniform body of fluid, which can be characterized in the usual way by a density $\rho$ and molecular coefficient of viscosity $\mu$. This body of fluid can be set into different kinds of motion. It is a well-known fact that under suitable conditions the kinematic viscosity $\upsilon$ be sufficiently small, some of these motions are such that the velocity at any given time and space in the fluid is not found to be the same. In these motions the velocity takes random values, which are not determined, although we believe that the average properties of the motion are determined uniquely. Fluctuating motions of this kind are said to be turbulent. Our concern is with homogeneous turbulence, which is a random motion whose average properties are independent of position in the fluid. The problem is to determine analytically the average properties and to understand the mechanics of this kind of motion.

The conception of homogeneous turbulence is idealized; there is no known method of realizing this kind of motion exactly. To produce turbulent motion in a laboratory or in nature we can apply various kinds of methods involving discrimination between different parts of the fluid, so that the average properties of the motion depend on position. From exact independence of position this departure can be made very small in certain circumstances and it is possible to get a close approximation to homogeneous turbulence. For instance, if a uniform stream of fluid passes through a regular array of holes in a rigid sheet, or a regular grid of bars, held at right angles to the stream, the motion downstream of the sheet consists of the same uniform velocity together with a superimposed random distribution of velocity. This random motion dies away with distance from the grid and is not statistically homogeneous, but the rate of decay is found to be so small that the assumption of homogeneity of the turbulence is valid for most purposes. A convenient laboratory method of producing turbulence is available there which is approximately homogeneous.

The kinds of turbulent motion which are encountered in nature, hydraulics and chemical engineering, are usually more complicated than homogeneous turbulence. These turbulent motions are such that there is a variation of the mean velocity with position in the first place and there is a variation of the average properties of the turbulent or fluctuating velocity with position in the second place. Thus there will occur some kind of interaction between the fluctuating and mean components of the motion which is difficult to handle mathematically and there will also be transport effects produced by the different intensity of the fluctuating motion at different points. As a preliminary, it seems appropriate to consider homogeneous turbulence that has neither of the two properties mentioned above.

In the simplest case, the turbulence is statistically homogeneous and isotropic so that depends neither the position nor the direction of the axis of reference. The possibility of this further assumption of isotropy exits only when the turbulence in already homogeneous. It has been found that in addition to being the simplest possible case of turbulent motion, isotropic turbulence is already generated in the laboratory. Whatever the initial directional properties of a field of homogeneous turbulence, it appears to settle down to an approximately isotropic state and the laboratory method of generating homogeneous turbulence by passing a uniform stream through a regular array, in fact turbulence which is very nearly isotropic and thus most of the available data concerns isotropic turbulence. However, sometimes there are some definite and important results for non-isotropic homogeneous turbulence.

Finally, if we collect the reasons for studying homogeneous turbulence, we should add that it is a interesting physical phenomenon which still defies satisfactory mathematical analysis. Again, if we study on isotropic turbulence then it is observed that this is the simplest type of turbulence, since no preference for any specific direction occurs and a minimum number of quantities and relations are required to describe its structure and behavior. However, it is also a hypothetical type of turbulence, because no actual turbulent flow shows true isotropy, though conditions may be made such that isotropy is more or less closely approached. The theoretical and experimental results of such a study of more practical value than one might believe. By theoretical considerations and experimental evidence it is known that the fine structure of most actual non-isotropic turbulent flows is nearly isotropic or simply local isotropy. So, many features of isotropic turbulence may apply to phenomena in actual turbulence that is determined mainly by the fine scale structure.

If we consider the non-isotropic turbulence through an essential part of its spectrum, it is often possible to treat such turbulence for purposes of a first approximation as if it were isotropic. Differences between results based upon the assumed isotropy and actual results are often sufficiently small to be negligible in a first approximation and may be even smaller than the spread in the experimental data. Because of its relative simplicity, isotropic turbulence has been studied most theoretically as well as experimentally. The kinematic and geometric relations involved in turbulence have been studied through correlations and spectrum functions.

## 1.4. Photographs of Turbulent Flow

The study on photographs of turbulent flows or on oscillograms of velocity fluctuations will reveal that it is not permissible to speak of the quasi periodicity or the scale of turbulence. Turbulence consists of many superimposed quasi-periodic motions. We also say that turbulence consists of the superposition of eddies of ever-smaller sizes, since a periodicity in velocity distribution involves the occurrence of velocity gradients which correspond to a certain vortex motion. All various-sized eddies of which a turbulent motion is composed have a certain kinetic energy determined by their vorticity or by the intensity of the velocity fluctuation of the corresponding frequency.

We have spoken about turbulent motion, which can be assumed to consist of the superposition of eddies of various sizes and vortices with distinguishable upper and lower limits. The upper size limit of the eddies is determined by the size of the apparatus, whereas the lower limit is determined mainly by viscous effects and decreases with increasing velocity of the average flow, other conditions remaining same. Within these smallest eddies the flow is no longer turbulent but viscous and molecular effects are dominant. These smallest eddies might not become so small that the flow within them could no longer be treated as a continuum flow.

For moderate flow velocities, i.e, not much greater than $100\ m/\sec$, say, the smallest space scale or eddy will hardly be less than about $1\ mm$. This value is still very large compared with the mean free path in gases under atmospheric conditions of order $10^{-4}\ mm$. Under atmospheric conditions, one cubic millimeter of air contains roughly $2.7\times10^{16}$ molecules. Thus under these atmospheric conditions, gases and liquids also may be treated in the study of turbulent flow of moderate speed. Relevant values of turbulent fluctuations are roughly 10 percent of average velocity and are between 1 and 1,000 $cm/\sec$. For air, these values must be compared with mean velocity of molecules of order $50{,}000\ cm/\sec$. Frequencies of turbulence vary from 1 and $10{,}000\ \sec^{-1}$, whereas molecular-collision frequencies for air are about $5\times10^{9}\ \sec^{-1}$.

From the domain of molecular magnitudes, the domain of turbulent magnitudes is sufficiently far. By discussing a few photographs of fluid motion we will conclude first introduction. The following figures help us to convey an idea about the size of these smallest eddies compared with the mean free path of the molecules. The general flow pattern is so regular that it hardly falls within the definition of real turbulence. At most this might be considered pseudo turbulence. The flow pattern just downstream of a circular cylinder at low values of the Reynolds number shows in Figure-1.6 [13]

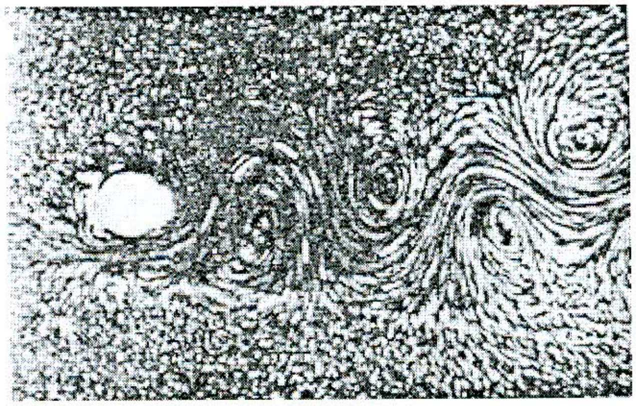


**Figure-1.6: Flow pattern downstream of a cylinder**

Figure-1.7 [13] shows a similar flow pattern, but one pertaining to a higher value of the Reynolds number. Up to downstream distances 30 to 40 times the cylinder diameter, the general flow pattern is still fairly regular; the more detailed patterns and beyond this distance, the general flow pattern also gradually become more and more turbulent.

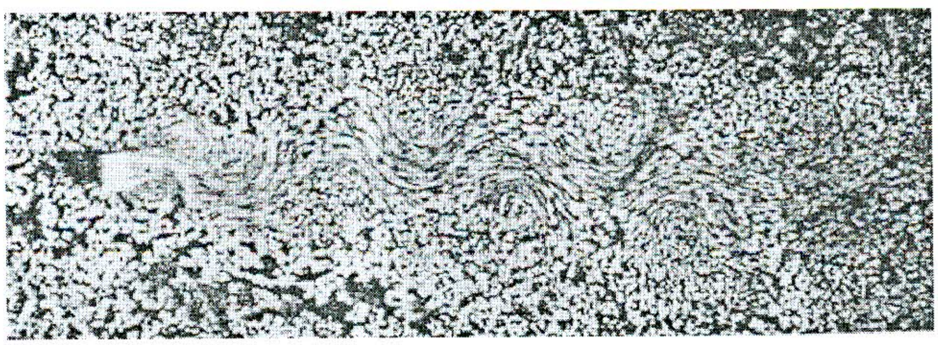


Figure-1.7: Flow pattern downstream of a cylinder

The detailed patterns become more turbulent as the Reynolds number increases is clearly shown in Figure-1.8 [13] which shows a close-up of the flow pattern close behind the cylinder. Within the region of the large regular eddies the flow pattern is distinctly turbulent; with a space scale much smaller than those of the large regular eddies. The regularity and irregularity of the flow in the wake of a cylinder are well illustrated by velocity oscillograms taken at different locations in the wake flow.

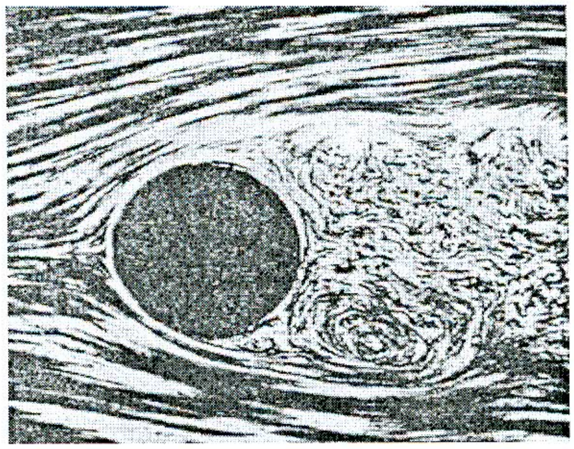


**Figure-1.8: Flow pattern close behind a cylinder at high Reynolds number**

An oscillogram taken at a point on the line through the centers of the vortices of each row shows a preference for a distinct frequency. Again an oscillogram taken centrally behind the cylinder also shows a preference for a distinct frequency equal to twice the previous frequency which is the effect of the vortices are separated from either side of the cylinder alternately.

## 1.5. Concepts on Fiber

The word 'Fiber' comes from Latin fibra. Various kinds of definitions of fiber are as follows:

A fiber is

i) a fine thread or thread-like cell of a natural such as cellulose, fruits, vegetables etc. or artificial substance such as nylon, cord, filament etc. Fibers composed of some natural and artificial substances, e.g. texture, grain, tissue, nap, grit, surface etc.

ii) the indigestible parts of edible plants or seeds that helps to move food quickly through the body. As for example, dietary fiber.

iii) one of the delicate, threadlike portions of which the tissues of plants and animals are in part constituted; as, the fiber of flax or of muscle.

iv) a general name for the raw material such as cotton, flax, hemp, etc., used in textile manufactures.

Fiber helps in the digestive process and is thought to lower cholesterol and help control blood glucose. There are two types of fiber in food. They are soluble and insoluble. The soluble fiber found in beans, fruits, and oat products, dissolves in water and is thought to help lower blood fats and blood glucose. Soluble fiber substances are effective in helping reduce the blood cholesterol. This is especially true with oat bran, fruits, psyllium and legumes. High soluble fiber diets may lower cholesterol and low-density lipoproteins by 8% to 15%. The insoluble fiber found in whole-grain products and vegetables, passes directly through the digestive system, helping to rid the body of waste products and possibly prevent diseases such as colon cancer. Insoluble fiber retains water in the colon, resulting in a softer and larger stool. It is used effectively in treating constipation resulting from poor dietary habits. Bran is particularly rich in insoluble fiber.

There are some useful fibers using in our daily life:

***Fiber Gun:*** A kind of steam gun for converting, wood, straw, etc., into fiber. The material is shut up in the gun with steam, air, or gas at a very high pressure, which is afterward relieved suddenly by letting a lid at the muzzle fly open, when the rapid expansion separates the fibers.

***Fiber Planets (Bot.):*** plants capable of yielding fiber useful in the arts, as hemp, flax, ramie etc.

***Optical Fiber:*** Optical fibre is a very thin fibre made of glass that functions as a waveguide for light; used in bundles to transmit images. Optical fibre is less susceptible to external noise than other transmission media, and is cheaper to make than copper wire, but it is much more difficult to connect.

***Fiber Channel:*** Serial data transfer architecture developed by a consortium of computer and mass storage device manufacturers. The most prominent Fiber Channel standard is Fiber Channel Arbitrated Loop, briefly (FC-AL). Fiber Channel was designed for new mass storage devices and other peripheral devices that require very high bandwidth.

# CHAPTER TWO
# Equations of Turbulent Motion

# CHAPTER TWO

# EQUATIONS OF TURBULENT MOTION

## 2.1. Introduction

We see that the physical nature of turbulence and its development is far from understandable; however, it can be studied both experimentally and theoretically as a statistical phenomenon. By this approach it is hoped that some important aspects of the structure of turbulent flows might be revealed. On the basis of these hypotheses, we have derived the equations of motion according as Reynolds, Navier-Stokes.

Osborn Reynolds derived an equation with viscosity and apparent or virtual stresses of turbulent flow or Reynolds stresses. Stresses are due to turbulent fluctuation and are given by the time mean values of the quadratic terms in the turbulent components. Since these stresses are added to the ordinary viscous terms in laminar flow and have a similar influence on the course of the flow, it is often said that they are caused by eddy viscosity. In vorticity transport theory, we take the conservation of vorticity instead of conservation of momentum so that the vorticity is taken to be transferable property of the fluid in this theory. The coriolis force due to rotation plays an important role for an equation of motion in a rotating system of turbulent flow, while the centrifugal force can be observed in the pressure gradient.

## 2.2. Reynolds Equation of Motion for Ordinary Turbulent Flow

To obtain the equations governing any turbulent flow, we take the Navier-Stokes equation of motion governing the flow and then take the time average of this equation.

For simplicity, we take the fluid to be incompressible, rather of constant density. The object is to derive the equations of motion that must be satisfied by the time averages of the velocity components $\bar{u}, \bar{v}, \bar{w}$ and of the pressure $\bar{p}$.

We consider the Navier-Stokes equation of motion for incompressible fluid with no body force,

$$\rho \frac{\partial \underline{v}}{\partial t} + \rho(\underline{v}.\nabla)\underline{v} = -\nabla p + \mu \nabla^2 \underline{v} \qquad (2.2.1)$$

The $x$-component of this equation is

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} \qquad (2.2.2)$$

Now using $u = \bar{u} + u', v = \bar{v} + v', w = \bar{w} + w', p = \bar{p} + p'$ in equation (2.2.2) we obtain-

$$\rho(\bar{u} + u')\frac{\partial}{\partial x}(\bar{u} + u') + \rho(\bar{v} + v')\frac{\partial}{\partial y}(\bar{u} + u') + \rho(\bar{w} + w')\frac{\partial}{\partial z}(\bar{u} + u') + \rho\frac{\partial}{\partial t}(\bar{u} + u')$$

$$= -\frac{\partial}{\partial x}(\bar{p} + p') + \mu\frac{\partial^2}{\partial x^2}(\bar{u} + u') \qquad (2.2.3)$$

We take the time average to each term of equation (2.2.3).

The first term on the left-hand side can be expressed as

$$\overline{\rho\left(\bar{u}+u'\right)\frac{\partial}{\partial x}\left(\bar{u}+u'\right)} = \overline{\rho\bar{u}\frac{\partial \bar{u}}{\partial x}} + \overline{\rho\bar{u}\frac{\partial u'}{\partial x}} + \overline{\rho u'\frac{\partial \bar{u}}{\partial x}} + \overline{\rho u'\frac{\partial u'}{\partial x}} \qquad (2.2.4)$$

The first term on the right-hand side of equation (2.2.4) cannot be changed by the time average, because the time average quantities remain constant in the interval $T$,

i.e, $$\overline{\rho\bar{u}\frac{\partial \bar{u}}{\partial x}} = \rho\bar{u}\frac{\partial \bar{u}}{\partial x}$$

In the second term, the time average of $\frac{\partial u'}{\partial x}$ is zero,

i.e, $$\overline{\rho\bar{u}\frac{\partial u'}{\partial x}} = 0$$

Similarly, the third term

$$\overline{\rho u'\frac{\partial \bar{u}}{\partial x}} = 0, \text{ since } \overline{\rho u'} = 0$$

The last term $\overline{\rho u'\frac{\partial u'}{\partial x}}$ is the product of two fluctuating components, its time average is not zero,

i.e, $$\overline{\rho u'\frac{\partial u'}{\partial x}} \neq 0$$

Thus equation (2.2.4) becomes

$$\overline{\rho\left(\bar{u}+u'\right)\frac{\partial}{\partial x}\left(\bar{u}+u'\right)} = \rho\bar{u}\frac{\partial \bar{u}}{\partial x} + \overline{\rho u'\frac{\partial u'}{\partial x}}$$

Similarly, $$\overline{\rho\left(\bar{v}+v'\right)\frac{\partial}{\partial y}\left(\bar{u}+u'\right)} = \rho\bar{v}\frac{\partial \bar{u}}{\partial y} + \overline{\rho v'\frac{\partial u'}{\partial y}}$$

and $$\overline{\rho\left(\bar{w}+w'\right)\frac{\partial}{\partial z}\left(\bar{u}+u'\right)} = \rho\bar{w}\frac{\partial \bar{u}}{\partial z} + \overline{\rho w'\frac{\partial u'}{\partial z}}$$

Since $\overline{\frac{\partial p'}{\partial x}} = 0, \overline{\frac{\partial u'}{\partial t}} = 0, \overline{\frac{\partial^2 u'}{\partial x^2}} = 0$ then

$$\overline{\rho \frac{\partial}{\partial t}\left(\bar{u} + u'\right)} = \rho \frac{\partial \bar{u}}{\partial t}$$

$$\overline{\frac{\partial}{\partial x}\left(\bar{p} + p'\right)} = \frac{\partial \bar{p}}{\partial x}$$

and $$\overline{\mu \frac{\partial^2}{\partial x^2}\left(\bar{u} + u'\right)} = \mu \frac{\partial^2 \bar{u}}{\partial x^2}$$

Hence equation (2.3) becomes

$$\rho \bar{u} \frac{\partial \bar{u}}{\partial x} + \overline{\rho u' \frac{\partial u'}{\partial x}} + \rho \bar{v} \frac{\partial \bar{u}}{\partial y} + \overline{\rho v' \frac{\partial u'}{\partial y}} + \rho \bar{w} \frac{\partial \bar{u}}{\partial z} + \overline{\rho w' \frac{\partial u'}{\partial z}} + \rho \frac{\partial \bar{u}}{\partial t} = -\frac{\partial \bar{p}}{\partial x} + \mu \frac{\partial^2 \bar{u}}{\partial x^2}$$

or, $$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z}\right) + \rho \frac{\partial \bar{u}}{\partial t} = -\frac{\partial \bar{p}}{\partial x} - \left(\overline{\rho u' \frac{\partial u'}{\partial x}} + \overline{\rho v' \frac{\partial u'}{\partial y}} + \overline{\rho w' \frac{\partial u'}{\partial z}}\right)$$

or, $$\rho \frac{\partial \bar{u}}{\partial t} + \rho \bar{u} \frac{\partial \bar{u}}{\partial x} + \rho \bar{v} \frac{\partial \bar{u}}{\partial y} + \rho \bar{w} \frac{\partial \bar{u}}{\partial z} + \overline{\rho u' \frac{\partial u'}{\partial x}} + \overline{\rho v' \frac{\partial u'}{\partial y}} + \overline{\rho w' \frac{\partial u'}{\partial z}}$$

$$= -\frac{\partial \bar{p}}{\partial x} + \mu \frac{\partial^2 \bar{u}}{\partial x^2} + \mu \frac{\partial^2 \bar{u}}{\partial x^2} \qquad (2.2.5)$$

It is verified that

$$-\left(\overline{\rho u' \frac{\partial u'}{\partial x}} + \overline{\rho v' \frac{\partial u'}{\partial y}} + \overline{\rho w' \frac{\partial u'}{\partial z}}\right) = \frac{\partial}{\partial x}\left(-\overline{\rho u'^2}\right) + \frac{\partial}{\partial y}\left(-\overline{\rho u'v'}\right) + \frac{\partial}{\partial z}\left(-\overline{\rho u'w'}\right)$$

Thus equation (2.2.5) becomes

$$\rho\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z}\right) = -\frac{\partial \bar{p}}{\partial x} + \mu \frac{\partial^2 \bar{u}}{\partial x^2}$$

$$+ \frac{\partial}{\partial x}\left(-\overline{\rho u'^2}\right) + \frac{\partial}{\partial y}\left(-\overline{\rho u'v'}\right) + \frac{\partial}{\partial z}\left(-\overline{\rho u'w'}\right) \qquad (2.2.6)$$

In similar way, $y$ and $z$ components of equation of motion are

$$\rho\left(\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{v}}{\partial x}+\bar{v}\frac{\partial \bar{v}}{\partial y}+\bar{w}\frac{\partial \bar{v}}{\partial z}\right)=-\frac{\partial \bar{p}}{\partial y}+\mu\frac{\partial^2 \bar{v}}{\partial y^2}$$

$$+\frac{\partial}{\partial x}\left(-\overline{\rho u'v'}\right)+\frac{\partial}{\partial y}\left(-\overline{\rho v'^2}\right)+\frac{\partial}{\partial z}\left(-\overline{\rho v'w'}\right) \qquad (2.2.7)$$

and

$$\rho\left(\frac{\partial \bar{w}}{\partial t}+\bar{u}\frac{\partial \bar{w}}{\partial x}+\bar{v}\frac{\partial \bar{w}}{\partial y}+\bar{w}\frac{\partial \bar{w}}{\partial z}\right)=-\frac{\partial \bar{p}}{\partial z}+\mu\frac{\partial^2 \bar{w}}{\partial z^2}$$

$$+\frac{\partial}{\partial x}\left(-\overline{\rho u'w'}\right)+\frac{\partial}{\partial y}\left(-\overline{\rho v'w'}\right)+\frac{\partial}{\partial z}\left(-\overline{\rho w'^2}\right) \qquad (2.2.8)$$

The additional terms over Navier-Stokes equation are called the Reynolds stress or eddy stress due to turbulent fluctuation in the flow field and the equations (2.2.6), (2.2.7) and (2.2.8) are known as Reynolds equation of motion for incompressible turbulent flow.

In vector tensor form, the nine eddy or Reynolds stress form the components of a second order tensor $T^{(e)}$ which is defined as

$$T^{(e)} = \begin{pmatrix} \sigma_{xx}^{(e)} & \tau_{xy}^{(e)} & \tau_{xz}^{(e)} \\ \tau_{yx}^{(e)} & \sigma_{yy}^{(e)} & \tau_{yz}^{(e)} \\ \tau_{zx}^{(e)} & \tau_{zy}^{(e)} & \sigma_{zz}^{(e)} \end{pmatrix}$$

$$= \begin{pmatrix} -\overline{\rho u'^2} & -\overline{\rho u'v'} & -\overline{\rho u'w'} \\ -\overline{\rho u'v'} & -\overline{\rho v'^2} & -\overline{\rho v'w'} \\ -\overline{\rho u'w'} & -\overline{\rho v'w'} & -\overline{\rho w'^2} \end{pmatrix}$$

where, $\sigma^{(e)} = -\rho\overline{u_i'^2}$ represents eddy normal stress and $\tau^{(e)} = -\rho\overline{u_i' u_j'}\,(i \neq j)$ represents eddy shearing stress.

The nine eddy stress terms are also called virtual stress of turbulent flow. Since, the stresses are added to the ordinary viscous terms and have a similar influence on the course of the flow so that it is said to be eddy viscosity. Therefore, the stress terms are added to the Reynolds equation may be written in the following form

$$\rho\left[\frac{\partial \underline{v}}{\partial t}+(\underline{v}.\nabla)\underline{v}\right]=-\nabla p+\mu\nabla^2\underline{v}+\left[\nabla.T^{(e)}\right]$$

or, $$\rho\frac{d\underline{v}}{dt}=-\nabla p+\mu\nabla^2\underline{v}+\left[\nabla.T^{(e)}\right]$$

which is the summarized form of the equation in vector tensor form.

## 2.3. Vorticity Equation for Ordinary Turbulent Flow

The equation of motion for incompressible fluid with viscous effect can be expressed as

$$\frac{d\underline{q}}{dt}=\underline{F}-\frac{1}{\rho}\nabla p+\upsilon\nabla^2\underline{q}$$

or, $$\frac{\partial \underline{q}}{\partial t}+(\underline{q}.\nabla)\underline{q}=\underline{F}-\frac{1}{\rho}\nabla p+\upsilon\nabla^2\underline{q} \qquad (2.3.1)$$

where, the symbols have their usual meaning;

In vector analysis,

$$\nabla(\underline{q}.\underline{q})=(\underline{q}.\nabla)\underline{q}+(\underline{q}.\nabla)\underline{q}+\underline{q}\times(\nabla\times\underline{q})+\underline{q}\times(\nabla\times\underline{q})$$

or, $$\nabla(\underline{q}.\underline{q})=2(\underline{q}.\nabla)\underline{q}+2\underline{q}\times(\nabla\times\underline{q})$$

or, $$(\underline{q}.\nabla)\underline{q}=\nabla\left(\frac{q^2}{2}\right)-\underline{q}\times(\nabla\times\underline{q})$$

Substituting the above expression in equation (2.3.1), we have

$$\frac{\partial \underline{q}}{\partial t} + \nabla\left(\frac{q^2}{2}\right) - \underline{q} \times \left(\nabla \times \underline{q}\right) = \underline{F} - \frac{1}{\rho}\nabla p + \upsilon \nabla^2 \underline{q} \qquad (2.3.2)$$

If the body force $\underline{F}$ is conservative then there exists a potential function $\varphi$ such that $\underline{F} = -\nabla \varphi$

Hence, equation (2.3.2) becomes

$$\frac{\partial \underline{q}}{\partial t} + \nabla\left(\frac{q^2}{2}\right) - \underline{q} \times \underline{\omega} = -\nabla \varphi - \frac{1}{\rho}\nabla p + \upsilon \nabla^2 \underline{q}$$

where, $\underline{\omega} = \nabla \times \underline{q}$ is the vorticity of the fluid;

Taking curl on both sides, we obtain

$$\frac{\partial}{\partial t}\left(\nabla \times \underline{q}\right) - \nabla \times \left(\underline{q} \times \underline{\omega}\right) = \upsilon \nabla^2 \left(\nabla \times \underline{q}\right)$$

or, $$\frac{\partial \underline{\omega}}{\partial t} - \nabla \times \left(\underline{q} \times \underline{\omega}\right) = \upsilon \nabla^2 \underline{\omega}$$

or, $$\frac{\partial \underline{\omega}}{\partial t} - \left[(\underline{\omega}.\nabla)\underline{q} + (\nabla.\underline{\omega})\underline{q} - (\nabla.\underline{q})\underline{\omega} - (\underline{q}.\nabla)\underline{\omega}\right] = \upsilon \nabla^2 \underline{\omega}$$

Since, $\nabla.\underline{\omega} = \nabla.\left(\nabla \times \underline{q}\right) = 0$ and for an incompressible fluid $\nabla.\underline{q} = 0$ then the above equation becomes

$$\frac{\partial \underline{\omega}}{\partial t} - (\underline{\omega}.\nabla)\underline{q} + (\underline{q}.\nabla)\underline{\omega} = \upsilon \nabla^2 \underline{\omega}$$

or, $$\frac{\partial \underline{\omega}}{\partial t} + (\underline{q}.\nabla)\underline{\omega} = (\underline{\omega}.\nabla)\underline{q} + \upsilon \nabla^2 \underline{\omega}$$

or, $$\frac{d\underline{\omega}}{dt} = (\underline{\omega}.\nabla)\underline{q} + \upsilon \nabla^2 \underline{\omega} \qquad (2.3.3)$$

This is known as the vorticity equation.

The first term on the right hand side of the equation (2.3.3) represents the

with the fluid, the strength of the vortex remaining constant. For slow motion, the term $(\underline{\omega}.\nabla)\underline{q}$ is negligible and the equation (2.3.3) becomes

$$\frac{d\underline{\omega}}{dt} = \upsilon \nabla^2 \underline{\omega} \tag{2.3.4}$$

This equation is called diffusion of vorticity.

In the case of two dimensional motion,

$$\underline{q} = u\underline{i} + v\underline{j}$$

Therefore, $\underline{\omega} = \nabla \times \underline{q} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ u & v & 0 \end{vmatrix} = \underline{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

Now, $(\underline{\omega}.\nabla)\underline{q} = \underline{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\left(\underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y}\right)\underline{q} = 0$

Thus for three-dimensional motion or two-dimensional motion, the equation (2.3.4) describes the way in which the vorticity is transmitted through a viscous fluid.

This equation resembles equation of heat conduction in a liquid. Thus, we conclude that vorticity diffuses through a liquid in such the same manner as heat does. So vorticity cannot be generated within the interior of a viscous fluid. It is infact transmitted from the boundaries into the fluid. As for example, a sailing ship will generates vortices in its way arising from a hull which is a moving boundary with the passage of time, the disturbance is soon damped out as the vortices diffuses through the water.

## 2.4. Equation of Motion in a Rotating System

The velocities and accelerations are measured referred to the Newton's second law of motion relative to an inertial axis. The rate of change of any vector is equal to its rate of change relative to the axes plus the rate of change due to rotation of axes. When the motion is referred to axes which rotate steadily, the coriolis force and centrifugal force must be supposed to act on the fluid. If $\Omega$ be the angular velocity then the centrifugal force may be written as $\underline{\Omega} \times (\underline{\Omega} \times \underline{r})$ which can be observed in the pressure gradient force. Here, we consider the ordinary turbulent flow in a rotating system. The main object is to derive an equation for ordinary turbulent flow as Navier-Stokes equation of motion in a rotating system.

For angular velocity $\underline{\Omega}$, a point with position vector $\underline{r}$ in the rotating system has the velocity $\underline{\Omega} \times \underline{r}$ as shown in the following figure:

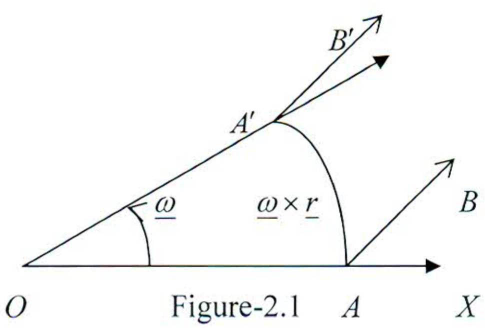



Figure-2.1

From the figure-2.1, when the particles goes from A to B relative to the axis $OX$ which rotates to $OX'$, at the same time the absolute replacement is therefore $A'B'$.

When the point $\underline{r}$ is moving relative rotating system, its velocity relative to the fixed system is given by

$$\left(\frac{d\underline{r}}{dt}\right)_f = \left(\frac{d\underline{r}}{dt}\right)_{rot} + \underline{\Omega}\times\underline{r} = \left(\frac{d}{dt} + \underline{\Omega}\times\right)\underline{r}$$

or,
$$\left(\frac{d\underline{r}}{dt}\right)_f = \frac{d\underline{r}'}{dt} + \underline{\Omega}\times\underline{r}$$

where, $\frac{d\underline{r}'}{dt}$ is the velocity of rotating system;

The acceleration is given by

$$\frac{d}{dt}\left(\frac{d\underline{r}}{dt}\right)_f = \left(\frac{d}{dt} + \underline{\Omega}\times\right)\left(\frac{d\underline{r}'}{dt} + \underline{\Omega}\times\underline{r}\right)$$

or,
$$\left(\frac{d^2\underline{r}}{dt^2}\right)_f = \frac{d}{dt}\left(\frac{d\underline{r}'}{dt}\right) + \frac{d}{dt}(\underline{\Omega}\times\underline{r}) + \underline{\Omega}\times\left(\frac{d\underline{r}'}{dt} + \underline{\Omega}\times\underline{r}\right)$$

$$= \left(\frac{d^2\underline{r}'}{dt^2}\right) + \underline{\Omega}\times\frac{d\underline{r}}{dt} + \underline{\Omega}\times(\underline{v} + \underline{\Omega}\times\underline{r})$$

$$= \frac{d^2\underline{r}'}{dt^2} + \underline{\Omega}\times\underline{v} + \underline{\Omega}\times\underline{v} + \underline{\Omega}\times(\underline{\Omega}\times\underline{r})$$

Thus,
$$\left(\frac{d^2\underline{r}}{dt^2}\right)_f = \frac{d^2\underline{r}'}{dt^2} + 2(\underline{\Omega}\times\underline{v}) + \underline{\Omega}\times(\underline{\Omega}\times\underline{r})$$

When $\underline{\Omega}$ is constant then the Navier-Stokes equation of motion in a rotating system gives
$$\frac{d\underline{v}}{dt} + 2(\underline{\Omega}\times\underline{v}) + \underline{\Omega}\times(\underline{\Omega}\times\underline{r}) = \underline{F} - \frac{1}{\rho}\nabla p + \upsilon\nabla^2\underline{v}$$

Since the centrifugal force can be absorbed in the pressure gradient, the Navier-Stokes equation of motion in a rotating system is given by

$$\frac{D\underline{v}}{Dt} + 2(\underline{\Omega} \times \underline{v}) = \underline{F} - \frac{1}{\rho}\nabla p + \upsilon\nabla^2 \underline{v}$$

Thus, the Coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force incorporated into the pressure.

## 2.5. Momentum and Induction Equation for MHD Turbulent Flow

We consider a conducting incompressible fluid for which there is no charge of accumulation at internal points. If we neglect the external force then the Navier-Stokes equation of motion can be expressed as

$$\rho\frac{d\underline{u}}{dt} = -\nabla p + \underline{J} \times \underline{B} + \rho\upsilon\nabla^2 \underline{u} \tag{2.5.1}$$

where, $\underline{u}$ is the velocity vector; $\rho$, the density of fluid particles; $p$, the pressure of the fluid; $\underline{B} = \mu\underline{H}$, the magnetic induction vector; $\underline{J}$, the current density vector; $\upsilon$, the kinematic coefficient of viscosity and $t$ is the time.

From Maxwell's relation, we have

$$\nabla \times \underline{H} = 4\pi\underline{J}$$

or, $$\underline{J} = \frac{1}{4\pi}(\nabla \times \underline{H})$$

Using these expressions in equation (2.5.1), we obtain

$$\rho \frac{d\underline{u}}{dt} = -\nabla p + \frac{\mu}{4\pi}(\nabla \times \underline{H}) \times \underline{H} + \rho \upsilon \nabla^2 \underline{u}$$

or, $$\frac{d\underline{u}}{dt} = -\frac{1}{\rho}\nabla p + \frac{\mu}{4\pi\rho}(\nabla \times \underline{H}) \times \underline{H} + \upsilon \nabla^2 \underline{u}$$

or, $$\frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\underline{u} = -\frac{1}{\rho}\nabla p + \frac{\mu}{4\pi\rho}(\nabla \times \underline{H}) \times \underline{H} + \upsilon \nabla^2 \underline{u} \qquad (2.5.2)$$

But in vector calculus, the term $(\nabla \times \underline{H}) \times \underline{H}$ can be written as

$$(\nabla \times \underline{H}) \times \underline{H} = -\nabla\left(\frac{H^2}{2}\right) + (\underline{H}.\nabla)\underline{H} \qquad (2.5.3)$$

Substituting equation $(2.5.3)$ in equation $(2.5.2)$, we get

$$\frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\underline{u} = -\frac{1}{\rho}\nabla p + \frac{\mu}{4\pi\rho}\left\{-\nabla\left(\frac{H^2}{2}\right) + (\underline{H}.\nabla)\underline{H}\right\} + \upsilon \nabla^2 \underline{u}$$

or, $$\frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\underline{u} - \frac{\mu}{4\pi\rho}(\underline{H}.\nabla)\underline{H} = -\frac{1}{\rho}\nabla p - \frac{\mu}{4\pi\rho}\nabla\left(\frac{H^2}{2}\right) + \upsilon \nabla^2 \underline{u} \qquad (2.5.4)$$

In the case of magneto-hydrodynamics, Chandra Sekhar extended invariant theory of turbulence. He introduced a variable defined as [9]

$$\underline{h} = \left(\frac{\mu}{4\pi\rho}\right)^{\frac{1}{2}} \underline{H}$$

If we use the variable in equation $(2.5.4)$ then we have

$$\frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\underline{u} - (\underline{h}.\nabla)\underline{h} = -\nabla\left(\frac{p}{\rho} + \frac{h^2}{2}\right) + \upsilon \nabla^2 \underline{u}$$

or, $$\frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\underline{u} - (\underline{h}.\nabla)\underline{h} = -\nabla p_n + \upsilon \nabla^2 \underline{u} \qquad (2.5.5)$$

where, $p_n = \frac{p}{\rho} + \frac{h^2}{2}$.

For the vector $\underline{u} = (u_1, u_2, u_3)$ and $\underline{h} = (h_1, h_2, h_3)$, in components form the equation $(2.5.5)$ is given by

$$\frac{\partial u_i}{\partial t} + (\underline{u}.\nabla)u_i - (\underline{h}.\nabla)h_i = -\frac{\partial p_n}{\partial x_i} + \upsilon\nabla^2 u_i \qquad (i = 1,2,3)$$

or, $$\frac{\partial u_i}{\partial t} + \sum_{k=1}^{3} u_k \frac{\partial u_i}{\partial x_k} - \sum_{k=1}^{3} h_k \frac{\partial h_i}{\partial x_k} = -\frac{\partial p_n}{\partial x_i} + \upsilon \sum_{k=1}^{3} \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

By using summation convention, the above equation becomes

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} - h_k \frac{\partial h_i}{\partial x_k} = -\frac{\partial p_n}{\partial x_i} + \upsilon \frac{\partial^2 u_i}{\partial x_k \partial x_k}$$

or, $$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial p_n}{\partial x_i} + \upsilon \frac{\partial^2 u_i}{\partial x_k \partial x_k} \qquad (2.5.6)$$

Equation $(2.5.6)$ is the momentum equation for incompressible MHD turbulent flow in components form in presence of viscous effects.

From magneto-hydrodynamics, the induction equation can be written as

$$\frac{\partial \underline{H}}{\partial t} = \nabla \times (\underline{u} \times \underline{H}) + \frac{1}{4\pi\sigma\mu}\nabla^2 \underline{H}$$

or, $$\frac{\partial \underline{H}}{\partial t} - \nabla \times (\underline{u} \times \underline{H}) = \lambda \nabla^2 \underline{H} \qquad (2.5.7)$$

where, $\lambda = (4\pi\sigma\mu)^{-1}$

From vector calculus, we have

$$\nabla \times (\underline{u} \times \underline{H}) = (\underline{H}.\nabla)\underline{u} + (\nabla.\underline{H})\underline{u} - (\underline{u}.\nabla)\underline{H} - (\nabla.\underline{u})\underline{H}$$

Therefore, equation $(2.5.7)$ becomes

$$\frac{\partial \underline{H}}{\partial t} - \{(\underline{H}.\nabla)\underline{u} + (\nabla.\underline{H})\underline{u} - (\underline{u}.\nabla)\underline{H} - (\nabla.\underline{u})\underline{H}\} = \lambda \nabla^2 \underline{H}$$

From Maxwell's relation, we write $\nabla.\underline{H} = 0$ and for an incompressible fluid, we obtain $\nabla.\underline{u} = 0$ so that the above equation gives

$$\frac{\partial \underline{H}}{\partial t} - \{(\underline{H}.\nabla)\underline{u} - (\underline{u}.\nabla)\underline{H}\} = \lambda \nabla^2 \underline{H} \tag{2.5.8}$$

Now, if we introduce the Chandra Sekhar variable $\underline{h} = \left(\frac{\mu}{4\pi\rho}\right)^{\frac{1}{2}} \underline{H}$ then the equation (2.5.8) becomes

$$\frac{\partial \underline{h}}{\partial t} + (\underline{u}.\nabla)\underline{h} - (\underline{h}.\nabla)\underline{u} = \lambda \nabla^2 \underline{h} \tag{2.5.9}$$

In components form,

the vectors $\underline{u} = (u_1, u_2, u_3)$

and $\underline{h} = (h_1, h_2, h_3)$,

Then the equation (2.5.9) is given by

$$\frac{\partial h_i}{\partial t} + (\underline{u}.\nabla)h_i - (\underline{h}.\nabla)u_i = \lambda \nabla^2 h_i \qquad (i = 1,2,3)$$

or, $$\frac{\partial h_i}{\partial t} + \sum_{k=1}^{3} u_k \frac{\partial h_i}{\partial x_k} - \sum_{k=1}^{3} h_k \frac{\partial u_i}{\partial x_k} = \lambda \sum_{k=1}^{3} \frac{\partial^2 h_i}{\partial x_k \partial x_k}$$

By using summation convention, the above equation becomes

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k}$$

or, $$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}\left(h_i u_k - u_i h_k\right)=\lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad (2.5.10)$$

Equation $(2.5.10)$ is the induction equation of incompressible fluid for MHD turbulent flow in components form in presence of viscous effects.

# CHAPTER THREE

# Fibre Suspensions in Turbulent Flow with Two-Point Correlation

# CHAPTER THREE

## Fiber Suspensions in Turbulent Flow with Two-Point Correlation

### 3.1. Introduction

The turbulent flow of fiber suspensions can be found in many areas of industry, such as the production of the composite materials, environmental engineering, chemical engineering, textile industry, paper making and so on. Fiber suspensions property has a significant effect on the quality of the products. So the fiber suspensions in a turbulent flow would be a better discussion. The interaction between the fluid and the fiber in a flow is complicated and it is more complicated if the flow is turbulent. The motion between a fluid particle and suspended fibers in order to behavior of turbulence with the correlations between pressure fluctuations and velocity fluctuations based on the basic fluid dynamics. The fiber orientation is an important physical quantity and do not only refer to rheology of fiber suspensions. Hinze [11] derived an expression for turbulent motion with the correlation between pressure fluctuations and velocity fluctuations at two points of the flow field. Anderson [2] discussed on some observation of fiber suspensions in turbulent motion. Batchelor [3] obtained the equations of motion of fiber suspensions in the flow. Zhang and Lin [32] studied on the motion of particles in the turbulent pipe flow of fiber suspensions. Lin et al [16] derived the new equation of turbulent fiber suspensions and its solution. They also verified the equations and their solutions by applying to a turbulent pipe flow of fiber suspensions. However, there are few studies relevant to the turbulent fiber suspension although it is prevalent in the industry. The main aim of this study is to derive an equation of motion for turbulent flow of fiber suspensions with two-point correlation between pressure fluctuations and velocity fluctuations.

### 3.2. Mathematical model of the problem

The equations of motion and continuity for turbulent flow of a viscous incompressible fluid are

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{3.2.1}$$

$$\frac{\partial u_i}{\partial x_i}=0 \tag{3.2.2}$$

For fiber suspensions into the flow, the equation of motion is given by [16]

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j \partial x_j}+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \tag{3.2.3}$$

where, $u_i$ are the fluid velocity components,

$p$ is the unknown pressure field,

$\upsilon$ is the kinematical viscosity of the suspending fluid,

$\mu_f$ is the apparent viscosity of fiber suspensions,

$\rho$ is the density of the fluid particle,

$\varepsilon_{lm} = \frac{1}{2}\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right)$ is the tensor of strain rate,

$I_{ij}$ is the turbulent intensity of suspensions,

$a_{lm}$ and $a_{ijlm}$ are the second and fourth-orientation tensors of the fiber respectively and t is the time.

We assume that the mean velocity $\overline{U_i}$ is constant throughout the region considered and independent of time and we put

$$\left(U_i = \overline{U}_i + u_i\right)_A,$$

$$\left(U_j = \overline{U}_j + u_j\right)_B.$$

The value of each term can be obtained by using the equations of motion for $u_j$ at the point $B$ and for $u_i$ at the point $A$. The equation of motion for $u_i$ at the point $A$, obtained from equation (3.2.3) takes the following form

$$\frac{\partial u_i}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon\frac{\partial^2 u_i}{\partial x_k \partial x_k} + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right] \qquad (3.2.4)$$

Since, for an incompressible fluid $\left(u_i \frac{\partial u_k}{\partial x_k}\right)_A = 0$, then the equation (3.2.4) can be written as

$$\frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A + \left(u_i \frac{\partial u_k}{\partial x_k}\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A$$

$$+ \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A \left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right]_A \qquad (3.2.5)$$

Multiplying equation (3.2.5) by $(u_j)_B$, we obtain

$$(u_j)_B \frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A (u_j)_B + (u_i)_A \left(\frac{\partial}{\partial x_k}\right)_A (u_k)_A (u_j)_B$$

$$= -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A (u_j)_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A (u_j)_B + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A \left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right]_A (u_j)_B \qquad (3.2.6)$$

Where, $(u_j)_B$ can be treated as a constant in a differential process at the point $A$.

Similarly, the equation of motion for $u_j$ at the point $B$ is given by

$$\frac{\partial u_j}{\partial t} + (\overline{U}_k + u_k)\frac{\partial u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \upsilon\frac{\partial^2 u_j}{\partial x_k \partial x_k} + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk}a_{lm})\varepsilon_{lm}\right] \qquad (3.2.7)$$

Since, for an incompressible fluid $\left(u_j \frac{\partial u_k}{\partial x_k}\right)_B = 0$ then the equation (3.2.7) can be written as

$$\frac{\partial}{\partial t}(u_j)_B + \left[\overline{U}_k + (u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B (u_j)_B + \left(u_j \frac{\partial u_k}{\partial x_k}\right)_B = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B (u_j)_B$$

$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B \left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk}a_{lm})\varepsilon_{lm}\right]_B \qquad (3.2.8)$$

Multiplying equation (3.2.8) by $(u_i)_A$, we get

$$(u_i)_A \frac{\partial}{\partial t}(u_j)_B + \left[\overline{U}_k + (u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B (u_j)_B (u_i)_A + (u_j)_B \left(\frac{\partial}{\partial x_k}\right)_B (u_k)_B (u_i)_A$$

$$= -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B (u_i)_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B (u_j)_B (u_i)_A$$

$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B \left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk}a_{lm})\varepsilon_{lm}\right]_B (u_i)_A \qquad (3.2.9)$$

where $(u_i)_A$ is a constant in a differential process at the point $B$.

Addition of the equations (3.2.6) and (3.2.9) gives the result

$$\frac{\partial}{\partial t}(u_i)_A(u_j)_B+\left[\left(\frac{\partial}{\partial x_k}\right)_A(u_i)_A(u_k)_A(u_j)_B+\left(\frac{\partial}{\partial x_k}\right)_B(u_i)_A(u_k)_B(u_j)_B\right]+\bar{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A(u_i)_A(u_j)_B+\left(\frac{\partial}{\partial x_k}\right)_B(u_i)_A(u_j)_B\right]$$

$$=-\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A(u_j)_B+\left(\frac{\partial}{\partial x_j}\right)_B p_B(u_i)_A\right]+\upsilon\left[\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A+\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B\right](u_i)_A(u_j)_B$$

$$+\frac{\mu_f}{\rho}\left[\left(\frac{\partial}{\partial x_k}\right)_A\left(a_{iklm}\varepsilon_{lm}-\frac{1}{3}I_{ik}a_{lm}\varepsilon_{lm}\right)_A(u_j)_B+\left(\frac{\partial}{\partial x_k}\right)_B\left(a_{jklm}\varepsilon_{lm}-\frac{1}{3}I_{jk}a_{lm}\varepsilon_{lm}\right)_B(u_i)_A\right] \quad (3.2.10)$$

To find the relation of turbulent fiber motions at the point $B$ to those at point $A$, it will give no difference if we take one point as the origin of $A$ or $B$ of the coordinate system. Let us consider the point $A$ as the origin. In order to differentiate between the effects of distance and location, we introduce as new independent variables

$$\zeta_k=(x_k)_B-(x_k)_A$$

Then we obtain, $\left(\frac{\partial}{\partial x_k}\right)_A=-\frac{\partial}{\partial\zeta_k},\ \left(\frac{\partial}{\partial x_k}\right)_B=\frac{\partial}{\partial\zeta_k}$

$$\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A=\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B=\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}$$

Using the above relations in equation (3.2.10) and taking ensemble average on both sides, we obtain

$$\frac{\partial}{\partial t}\overline{(u_i)_A(u_j)_B}-\frac{\partial}{\partial\zeta_k}\overline{(u_i)_A(u_k)_A(u_j)_B}+\frac{\partial}{\partial\zeta_k}\overline{(u_i)_A(u_k)_B(u_j)_B}=-\frac{1}{\rho}\left[-\frac{\partial}{\partial\zeta_i}\overline{p_A(u_j)_B}+\frac{\partial}{\partial\zeta_j}\overline{p_B(u_i)_A}\right]$$

$$+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}\overline{(u_i)_A(u_j)_B}-\frac{\mu_f}{\rho}\frac{\partial}{\partial\zeta_k}\left[\overline{(a_{iklm}\varepsilon_{lm})_A(u_j)_B}-\frac{1}{3}\overline{(I_{ik}a_{lm}\varepsilon_{lm})_A(u_j)_B}\right]$$

$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial\zeta_k}\left[\overline{(a_{jklm}\varepsilon_{lm})_B(u_i)_A}-\frac{1}{3}\overline{(I_{jk}a_{lm}\varepsilon_{lm})_B(u_i)_A}\right] \quad (3.2.11)$$

This equation represents the equation of mean motion of fiber suspensions in turbulent flow with pressure-velocity correlation.

It is clear that the coefficient of $\overline{U}_k$ has been vanished. The equation (3.2.11) describes the turbulent motion of fiber suspensions, where the motions with respect to a coordinate system moving with the mean velocity $\overline{U}_k$.

Equation (3.2.11) contains the double velocity correlation $\overline{(u_i)_A(u_j)_B}$, double correlations such as $\overline{p_A(u_j)_B}$, triple correlations such as $\overline{(u_i)_A(u_k)_A(u_j)_B}$ where all the terms apart from one another. The correlations $\overline{p_A(u_j)_B}$ and $\overline{p_B(u_i)_A}$ form the tensors of first order, because pressure is a scalar quantity and the triple correlations $\overline{(u_i)_A(u_k)_A(u_j)_B}$ and $\overline{(u_i)_A(u_k)_B(u_j)_B}$ form the tensors of third order.

We designate the first order correlations by $(k_{p,j})_{A,B}$, second order correlations by $(Q_{i,j})_{A,B}$ and third order correlations by $(s_{ik,j})_{A,B}$.

Therefore, we set

$$(k_{i,p})_{A,B} = \overline{(u_i)_A p_B},\ (k_{p,j})_{A,B} = \overline{p_A(u_j)_B},$$

$$(Q_{i,j})_{A,B} = \overline{(u_i)_A(u_j)_B},$$

$$(s_{ik,j})_{A,B} = \overline{(u_i)_A(u_k)_A(u_j)_B},$$

$$(s_{i,kj})_{A,B} = \overline{(u_i)_A(u_k)_B(u_j)_B},$$

Where, the index $p$ indicates the pressure and is not a dummy index like $i$ or $j$ so that the summation convention does not apply to $p$.

Also the term $\overline{(a_{jklm}\varepsilon_{lm})_B(u_i)_A}$ and $\overline{(I_{jk}a_{lm}\varepsilon_{lm})_B(u_i)_A}$ form the correlation tensors of third order, we designate these by $D_{i,jk}$ and $H_{i,jk}$ respectively.

Thus we set $\left(D_{i,jk}\right)_{A,B}=\overline{\left(u_i\right)_A\left(a_{jklm}\varepsilon_{lm}\right)_B}$,

$$\left(D_{ik,j}\right)_{A,B}=\overline{\left(a_{iklm}\varepsilon_{lm}\right)_A\left(u_j\right)_B},$$

$$\left(H_{i,jk}\right)_{A,B}=\overline{\left(u_i\right)_A\left(I_{jk}a_{lm}\varepsilon_{lm}\right)_B},$$

$$\left(H_{ik,j}\right)_{A,B}=\overline{\left(I_{ik}a_{lm}\varepsilon_{lm}\right)_A\left(u_j\right)_B}.$$

If we use the above relations of first, second and third order correlations in equation (3.2.11), then we obtain

$$\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial\zeta_k}S_{ik,j}+\frac{\partial}{\partial\zeta_k}S_{i,kj}=\frac{1}{\rho}\left(-\frac{\partial}{\partial\zeta_i}K_{p,j}+\frac{\partial}{\partial\zeta_j}K_{i,p}\right)+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}$$
$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial\zeta_k}\left[\left(D_{i,jk}-D_{ik,j}\right)+\frac{1}{3}\left(H_{ik,j}-H_{i,jk}\right)\right] \qquad (3.2.12)$$

where all the correlations refer to the two points $A$ and $B$.

Now for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero,

that is,

$$\left(k_{p,j}\right)_{A,B}=0,\ \left(k_{i,p}\right)_{A,B}=0.$$

In an isotropic turbulence it follows from the condition of invariance under reflection with respect to point $A$,

$$\overline{\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B}=-\overline{\left(u_k\right)_A\left(u_j\right)_A\left(u_i\right)_B}$$

or, $\left(s_{i,kj}\right)_{A,B}=-\left(s_{kj,i}\right)_{A,B}$

and hence $\left(D_{i,jk}\right)_{A,B}=-\left(D_{jk,i}\right)_{A,B}$,

$$\left(H_{i,jk}\right)_{A,B}=-\left(H_{jk,i}\right)_{A,B}.$$

After substitution these above relations, equation (3.2.12) becomes

$$\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial\zeta_k}\left(S_{ik,j}+S_{kj,i}\right)=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}$$

$$+\frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial\zeta_k}\left(D_{jk,i}+D_{ik,j}\right)+\frac{1}{3}\frac{\partial}{\partial\zeta_k}\left(H_{ik,j}+H_{jk,i}\right)\right] \qquad (3.2.13)$$

The term $\frac{\partial}{\partial\zeta_k}\left(S_{ik,j}+S_{kj,i}\right)$, $\frac{\partial}{\partial\zeta_k}\left(D_{jk,i}+D_{ik,j}\right)$ and $\frac{\partial}{\partial\zeta_k}\left(H_{ik,j}+H_{jk,i}\right)$ form the tensors of second order, we designate these by $S_{i,j}$, $D_{i,j}$ and $H_{i,j}$ respectively, that is

$$S_{i,j}=\frac{\partial}{\partial\zeta_k}\left(S_{ik,j}+S_{kj,i}\right),\ D_{i,j}=\frac{\partial}{\partial\zeta_k}\left(D_{jk,i}+D_{ik,j}\right)$$

and $H_{i,j}=\frac{\partial}{\partial\zeta_k}\left(H_{ik,j}+H_{jk,i}\right)$.

Therefore equation (3.2.13) gives the result

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}-\frac{\mu_f}{\rho}\left(D_{i,j}-\frac{1}{3}H_{i,j}\right) \qquad (3.2.14)$$

Equation (3.2.14) is the equation of motion for turbulent flow of fiber suspensions in terms of correlation tensors of second order.

If there are no effects of fiber suspension in the flow then the apparent viscosity of the fluid vanishes, that is, $\mu_f=0$ so that the equation (3.2.14) takes the form

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j} \qquad (3.2.15)$$

This equation represents the turbulent motion in terms of correlation tensors of second order, which is the same as obtained by Hinze [11].

### 3.3. Discussion and Conclusion

The equation of motion for turbulent flow of fiber suspensions has been derived by averaging procedure, which includes the effect of fiber suspensions and the correlation between the pressure fluctuations and velocity fluctuations at two points of the fluid flow. The discussion provides the equation of fiber mean motion, as well as for the resulting turbulent fiber motion. The interaction between the turbulent fluid and the fiber based on the Reynolds number. The occurrence of the turbulent flow will depend on the values of the non-dimensional number known as critical Reynolds number, which varies from 2000 to 2300. The flow will be turbulent if the Reynolds number $(\mathrm{Re})$ is greater than the critical Reynolds number $(R_{cr})$, so that the turbulent flow occurs at high Reynolds number. If the Reynolds number increases from 1600 to 2500 then the flow converts to turbulent flow from laminar flow, the orientation distribution of fiber changes in a range. It is clear that turbulence has effect on the orientation distribution of fiber.

Fiber suspensions in a turbulent fluid undergo mean motion due to the mean fluid velocity and random motion due to the fluctuating component of fluid velocity. The velocity of fiber fluctuates around the mean velocity of flow. Fluctuation velocity of turbulence at the two points $A$ and $B$ of the flow field leads to a weakening of the concentration of the fiber orientation distribution on small angle. This concentration leads to be weaker and orientation distribution of fiber becomes more uniform as Reynolds numbers increases and flow fluctuation velocity strengths. The velocity of fiber has the same fluctuation property as fluid velocity due to the strong following ability of fiber. The fluctuation velocity of fiber on flow direction is more energetic than that on lateral direction. As Reynolds number increases, the intensity of fluctuation velocity enhances, flow velocity gradient becomes more irregular and orientation distribution of fiber becomes wider. Thus the resulting equation demonstrates that as Reynolds number increases, the fluctuation velocity of turbulence at two points in the flow field becomes to be weaker, fiber orientation distribution tends to be more uniform and fluctuation velocity of fluid flow strengthens.

# CHAPTER FOUR

# Fibre Motion in Dusty Fluid Turbulent Flow with Two-point Correlation

# CHAPTER FOUR

## Fiber Motion in Dusty Fluid Turbulent Flow with Two-point Correlation

### 4.1. Introduction

The behavior of dust particles in a turbulent fluid depends on the concentration of the particles and on the size of the particles with respect to the scale of turbulent fluid. A fiber suspension in a turbulent flow affect the transport, rheology and light scattering properties of suspensions that are of great interest in many areas of science and industry. At great concentration there is an interaction between the particles through collisions and through the effects on the flow of the fluid in the neighborhood of the particles. Hinze [11] obtained an expression for correlation between pressure fluctuations and velocity fluctuations in turbulent motion. Saffman [22] observed the effect of dust particles of an incompressible flow and derived an equation that described the motion of a fluid containing small dust particles. Anderson [2] discussed on some observation of fiber suspensions in turbulent motion. Batchelor [7] obtained the equations of motion of fiber suspensions in the flow. Agermann and kohler [1] studied on rotational and translational dispersion of fibers in turbulent flow by assuming the dimension of fibers to be less than that of smallest eddies in the flow. Kishore and Abhilasha Sinha [13] derived an analytical expression for the rate of change of vorticity covariance in dusty fluid turbulent flow. Lin et al [16] derived the new equation of turbulent fiber suspensions and its solution and application to the pipe flow. The main aim of this study is to derive an equation for fiber motion in dusty fluid turbulent flow with the aid of pressure-velocity correlation.

## 4.2. Mathematical Model of the Problem

Let us assume that the fluid is to be incompressible. The equation of motion and continuity for fiber suspensions in turbulent flow of viscous incompressible fluid are [16]

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j \partial x_j}+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \quad (4.2.1)$$

$$\frac{\partial u_i}{\partial x_i}=0 \quad (4.2.2)$$

In presence of dust particles the equations of motion are given by

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j \partial x_j}+\frac{KN}{\rho}(v_i-u_i)+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \quad (4.2.3)$$

$$\frac{\partial u_i}{\partial x_i}=0 \quad (4.2.4)$$

$$\frac{\partial v_i}{\partial t}+v_j\frac{\partial u_i}{\partial x_j}=-\frac{K}{m_s}(u_i-v_i) \quad (4.2.5)$$

where $u_i(x,t)$ is the fluid velocity components,

$v_i(x,t)$, the solid particles (dust) velocity components

$p(x,t)$, the unknown pressure field

$m_s=\frac{4}{3}\pi R_s^3\rho_s$, the mass of a single spherical dust particles of radius $R_s$

$\upsilon=$constant is the molecular kinematic viscosity

$K=6\pi R_s\rho v$, the Stoke's drag formula

$N$, the number density of dust particles

$\frac{KN}{\rho} = f$, has dimension of frequency

$\mu_f$, the apparent viscosity of fiber suspensions

$\rho$, the density of the fluid particle

$\varepsilon_{lm} = \frac{1}{2}\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right)$ is the tensor of strain rate

$I_{ij}$, the turbulent intensity of suspensions

$a_{lm}$ and $a_{ijlm}$ are the second and fourth-orientation tensors of the fiber respectively

and t is the time.

We assume that the mean velocity $\overline{U_i}$ is constant throughout the region considered and independent of time and we put

$$\left(U_i = \overline{U}_i + u_i\right)_A,$$

$$\left(U_j = \overline{U}_j + u_j\right)_B.$$

The value of each term can be obtained by taking the equations of motion for $u_j$ at the point $B$ and for $u_i$ at the point $A$.

The equation of motion for $u_i$ at the point $A$, obtain from equation (4.2.3),

$$\frac{\partial u_i}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon\frac{\partial^2 u_i}{\partial x_k \partial x_k} + f\left(v_i - u_i\right)$$

$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}\left(I_{ik}a_{lm}\right)\varepsilon_{lm}\right] \qquad (4.2.6)$$

For an incompressible fluid $\left(u_i \dfrac{\partial u_k}{\partial x_k}\right)_A = 0$ so that equation (4.2.6) can be written as

$$\frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A + \left(u_i \frac{\partial u_k}{\partial x_k}\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A$$

$$+ f(v_i - u_i)_A + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A \left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right]_A \quad (4.2.7)$$

Multiply equation (4.2.7) by $(u_j)_B$ we obtain

$$(u_j)_B \frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A (u_j)_B + (u_i)_A\left(\frac{\partial}{\partial x_k}\right)_A (u_k)_A (u_j)_B$$

$$= -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A (u_j)_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A (u_j)_B + f(v_i - u_i)_A (u_j)_B$$

$$+ \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A \left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right]_A (u_j)_B \quad (4.2.8)$$

where $(u_j)_B$ can be treated as a constant in a differential process at the point $A$.

Similarly, the equation of motion for $u_j$ at the point $B$,

$$\frac{\partial u_j}{\partial t} + (\overline{U}_k + u_k)\frac{\partial u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \upsilon\frac{\partial^2 u_j}{\partial x_k \partial x_k} + f(v_j - u_j) + \frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk}a_{lm})\varepsilon_{lm}\right]$$

Since, for an incompressible fluid $\left(u_j \dfrac{\partial u_k}{\partial x_k}\right)_B = 0$ then the above equation can be written as

$$\frac{\partial}{\partial t}(u_j)_B + \left[\overline{U}_k + (u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B (u_j)_B + \left(u_j \frac{\partial u_k}{\partial x_k}\right)_B = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B (u_j)_B$$

$$+ f(v_j - u_j)_B + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B \left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk}a_{lm})\varepsilon_{lm}\right]_B \quad (4.2.9)$$

Multiplying equation (4.2.9) by $(u_i)_A$, we get

$$(u_i)_A \frac{\partial}{\partial t}(u_j)_B + \left[\overline{U}_k + (u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B (u_j)_B (u_i)_A + (u_j)_B \left(\frac{\partial}{\partial x_k}\right)_B (u_k)_B (u_i)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B (u_i)_A$$

$$+\upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B (u_j)_B (u_i)_A + f(v_j - u_j)_B (u_i)_A + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B \left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk}a_{lm})\varepsilon_{lm}\right]_B (u_i)_A \quad (4.2.10)$$

where $(u_i)_A$ can be treated as a constant in a differential process at the point $B$.

Addition of the equations (4.2.8) and (4.2.10) give the result

$$\frac{\partial}{\partial t}(u_i)_A (u_j)_B + \left[\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A (u_k)_A (u_j)_B + \left(\frac{\partial}{\partial x_k}\right)_B (u_i)_A (u_k)_B (u_j)_B\right] + \overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A (u_j)_B\right]$$

$$+\overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_B (u_i)_A (u_j)_B\right] = -\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A (u_j)_B + \left(\frac{\partial}{\partial x_j}\right)_B p_B (u_i)_A\right]$$

$$+ f\left[(v_i - u_i)_A (u_j)_B + (v_j - u_j)_B (u_i)_A\right] + \upsilon\left[\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A + \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\right](u_i)_A (u_j)_B$$

$$+\frac{\mu_f}{\rho}\left[\left(\frac{\partial}{\partial x_k}\right)_A \left(a_{iklm}\varepsilon_{lm} - \frac{1}{3}I_{ik}a_{lm}\varepsilon_{lm}\right)_A (u_j)_B + \left(\frac{\partial}{\partial x_k}\right)_B \left(a_{jklm}\varepsilon_{lm} - \frac{1}{3}I_{jk}a_{lm}\varepsilon_{lm}\right)_B (u_i)_A\right] \quad (4.2.11)$$

To find the relation of turbulent fiber motions in presence of dust particles at the point $B$ to those at point $A$, it will give no difference if we take one point as the origin of $A$ or $B$ of the coordinate system. Let us consider the point $A$ as the origin and can write

$$\zeta_k = (x_k)_B - (x_k)_A$$

Then we obtain,
$$\left(\frac{\partial}{\partial x_k}\right)_A = -\frac{\partial}{\partial \zeta_k}, \left(\frac{\partial}{\partial x_k}\right)_B = \frac{\partial}{\partial \zeta_k}$$

$$\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A = \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B = \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}$$

Using the above relations and taking ensemble average on both sides, equation (4.2.11) becomes

$$\frac{\partial}{\partial t}\overline{(u_i)_A(u_j)_B} - \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A(u_k)_A(u_j)_B} + \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A(u_k)_B(u_j)_B} = -\frac{1}{\rho}\left[-\frac{\partial}{\partial \zeta_i}\overline{p_A(u_j)_B} + \frac{\partial}{\partial \zeta_j}\overline{p_B(u_i)_A}\right]$$

$$-\frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\overline{(a_{iklm}\varepsilon_{lm})_A(u_j)_B} - \frac{1}{3}\overline{(I_{ik}a_{lm}\varepsilon_{lm})_A(u_j)_B} - \overline{(a_{jklm}\varepsilon_{lm})_B(u_i)_A} + \frac{1}{3}\overline{(I_{jk}a_{lm}\varepsilon_{lm})_B(u_i)_A}\right]$$

$$+2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}\overline{(u_i)_A(u_j)_B} + f\left[\overline{(v_i)_A(u_j)_B} - 2\overline{(u_i)_A(u_j)_B} + \overline{(u_i)_A(v_j)_B}\right] \quad (4.2.12)$$

The equation (4.2.12) represents the mean motion equation of fiber suspensions in turbulent flow in presence of dust particles and the pressure-velocity correlation.

It is noted that the coefficient of $\overline{U}_k$ has been vanished. The equation (4.2.12) describes the motions of fiber suspensions in turbulent flow in presence of dust particles, where the motions with respect to a coordinate system moving with the mean velocity $\overline{U}_k$. Equation (4.2.12) contains the double velocity correlation $\overline{(u_i)_A(u_j)_B}$, double velocity correlation between dust particles and the fluid such as $\overline{(v_i)_A(u_j)_B}$, double correlations such as $\overline{p_A(u_j)_B}$, triple correlations such as $\overline{(u_i)_A(u_k)_A(u_j)_B}$ where all the terms apart from one another. The correlations $\overline{p_A(u_j)_B}$ and $\overline{p_B(u_i)_A}$ form the tensors of the first order, because pressure is a scalar quantity and the triple correlations $\overline{(u_i)_A(u_k)_A(u_j)_B}$ and $\overline{(u_i)_A(u_k)_B(u_j)_B}$ form the tensors of third order.

We designate the first order correlations by $(k_{p,j})_{A,B}$, second order correlations by $(Q_{i,j})_{A,B}$ and third order correlations by $(s_{ik,j})_{A,B}$.

Therefore, we set $(k_{i,p})_{A,B} = \overline{(u_i)_A p_B}$, $(k_{p,j})_{A,B} = \overline{p_A(u_j)_B}$,

$$(s_{ik,j})_{A,B} = \overline{(u_i)_A (u_k)_A (u_j)_B},$$

$$(s_{i,kj})_{A,B} = \overline{(u_i)_A (u_k)_B (u_j)_B},$$

$$(Q_{i,j})_{A,B} = \overline{(u_i)_A (u_j)_B}, (F_{i,j})_{A,B} = \overline{(v_i)_A (u_j)_B},$$

and $(G_{i,j})_{A,B} = \overline{(u_i)_A (v_j)_B}$.

where the index $p$ indicates the pressure and is not a dummy index like $i$ or $j$ so that the summation convention does not apply to $p$.

Also the term $\overline{(a_{jklm}\varepsilon_{lm})_B (u_i)_A}$ and $\overline{(I_{jk}a_{lm}\varepsilon_{lm})_B (u_i)_A}$ form the correlations of third order, we designate these by $D_{i,jk}$ and $H_{i,jk}$ respectively.

Thus we set $(D_{i,jk})_{A,B} = \overline{(u_i)_A (a_{jklm}\varepsilon_{lm})_B},$

$$(D_{ik,j})_{A,B} = \overline{(a_{iklm}\varepsilon_{lm})_A (u_j)_B},$$

$$(H_{i,jk})_{A,B} = \overline{(u_i)_A (I_{jk}a_{lm}\varepsilon_{lm})_B},$$

$$(H_{ik,j})_{A,B} = \overline{(I_{ik}a_{lm}\varepsilon_{lm})_A (u_j)_B}.$$

If we use the above relations of first, second and third order correlations in equation (4.2.12) then we obtain

$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}S_{ik,j} + \frac{\partial}{\partial \zeta_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j} + \frac{\partial}{\partial \zeta_j}K_{i,p}\right) + 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$

$$+ f(F_{i,j} - 2Q_{i,j} + G_{i,j}) + \frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial \zeta_k}\left(D_{ik,j} - \frac{1}{3}H_{ik,j}\right) + \frac{\partial}{\partial \zeta_k}\left(D_{i,jk} - \frac{1}{3}H_{i,jk}\right)\right]$$

or, $$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}S_{ik,j} + \frac{\partial}{\partial \zeta_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j} + \frac{\partial}{\partial \zeta_j}K_{i,p}\right) + 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$

$$+ f(F_{i,j} - 2Q_{i,j} + G_{i,j}) + \frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[(D_{i,jk} - D_{ik,j}) + \frac{1}{3}(H_{ik,j} - H_{i,jk})\right] \quad (4.2.13)$$

where all the correlations refer to the two points $A$ and $B$.

Now for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero, that is

$$\left(k_{p,j}\right)_{A,B}=0,\left(k_{i,p}\right)_{A,B}=0$$

In an isotropic turbulence it follows from the condition of invariance under reflection with respect to point $A$,

$$\overline{(u_i)_A(u_k)_B(u_j)_B}=-\overline{(u_k)_A(u_j)_A(u_i)_B}$$

or

$$\left(s_{i,kj}\right)_{A,B}=-\left(s_{kj,i}\right)_{A,B}$$

and hence $\left(D_{i,jk}\right)_{A,B}=-\left(D_{jk,i}\right)_{A,B},\left(H_{i,jk}\right)_{A,B}=-\left(H_{jk,i}\right)_{A,B}$.

Thus equation (4.2.13) can be written as

$$\frac{\partial}{\partial t}Q_{i,j}-\frac{\partial}{\partial\zeta_k}\left(S_{ik,j}+S_{kj,i}\right)=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}+f\left(F_{i,j}-2Q_{i,j}+G_{i,j}\right)$$

$$+\frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial\zeta_k}\left(D_{jk,i}+D_{ik,j}\right)+\frac{1}{3}\frac{\partial}{\partial\zeta_k}\left(H_{ik,j}+H_{jk,i}\right)\right]\quad(4.2.14)$$

The term $\frac{\partial}{\partial\zeta_k}\left(S_{ik,j}+S_{kj,i}\right)$, $\frac{\partial}{\partial\zeta_k}\left(D_{jk,i}+D_{ik,j}\right)$ and $\frac{\partial}{\partial\zeta_k}\left(H_{ik,j}+H_{jk,i}\right)$ form the tensors of second order, we designate these by $S_{i,j},D_{i,j}$ and $H_{i,j}$ respectively, that is

$$S_{i,j}=\frac{\partial}{\partial\zeta_k}\left(S_{ik,j}+S_{kj,i}\right),D_{i,j}=\frac{\partial}{\partial\zeta_k}\left(D_{jk,i}+D_{ik,j}\right)\text{ and }H_{i,j}=\frac{\partial}{\partial\zeta_k}\left(H_{ik,j}+H_{jk,i}\right).$$

Therefore equation (4.2.14) gives the result

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}+f\left(F_{i,j}-2Q_{i,j}+G_{i,j}\right)-\frac{\mu_f}{\rho}\left(D_{i,j}-\frac{1}{3}H_{i,j}\right)\quad(4.2.15)$$

This is the equation of motion of fiber suspensions in dusty fluid turbulent flow in terms of correlation tensors of second order.

In absence of dust particles, $f = 0$ then equation (4.2.15) reduces to

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} - \frac{\mu_f}{\rho}\left(D_{i,j} - \frac{1}{3}H_{i,j}\right) \tag{4.2.16}$$

The equation (4.2.16) describes the turbulent motion of fiber suspensions in terms of the correlation tensors of second order.

If there are no effects of fiber suspension in the flow field then $\mu_f = 0$, so that the equation (4.2.16) takes the form

$$\frac{\partial}{\partial t}Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} \tag{4.2.17}$$

The equation (4.2.17) represents the turbulent motion in terms of correlation tensors of second order which is the same as obtained by J. O. Hinze.

### 4.3. Discussion and Conclusion

The equation of motion of fiber suspensions in dusty turbulent flow has been presented by taking the average procedure, which includes the effect of dust particles and the correlations between the pressure fluctuations and velocity fluctuations at two points of the flow field. The discussion provides the equation of fiber mean motion, as well as for the resulting dusty turbulent fiber motion. The interaction between the dusty turbulent fluid and the fiber based on the Reynolds number. The occurrence of the turbulent flow will depend on the values of the non-dimensional number known as critical Reynolds number, which varies from 2000 to 2300. The flow will be turbulent if the Reynolds number $(R_e)$ is greater than the critical Reynolds number $(R_{cr})$, so that the turbulent flow occurs at high Reynolds number. If the Reynolds number increases from 1600 to 2500 then the flow converts to turbulent flow from laminar flow, the orientation distribution of fiber changes in a range. It is clear that turbulence has effect on the orientation distribution of fiber and dust particles.

Fiber suspensions in a turbulent fluid undergo mean motion due to the mean fluid velocity and random motion due to the fluctuating component of fluid velocity. The velocity of fiber fluctuates around the mean velocity of flow. Under the influence of dust particles, fluctuation velocity of turbulence at the two points $A$ and $B$ of the flow field leads to a weakening of the concentration of the fiber orientation distribution on small angle. In presence of dust particles this concentration leads to be weaker and orientation distribution of fiber becomes more uniform as Reynolds numbers increases. The velocity of fiber has the same fluctuation property as fluid velocity due to the strong following ability of fiber. The fluctuation velocity of fiber on flow direction is more energetic than that on lateral direction. As Reynolds number increases, the intensity of fluctuation velocity enhances, flow velocity gradient becomes more irregular and orientation distribution of fiber becomes wider. Thus the resulting equation demonstrates that as Reynolds number increases, in presence of dust particles the fluctuation velocity of turbulence at two points in the flow field becomes to be weaker and fiber orientation distribution tends to be more uniform.

# CHAPTER FIVE

# Motion of Fibres in Turbulent Flow in a Rotating System

# CHAPTER FIVE

## Motion of Fibers in Turbulent Flow in a Rotating System

### 5.1. Introduction

The dynamics of fiber suspension depends heavily on the nature and magnitude of the fiber-fiber interactions. Long range and short range hydrodynamic interactions between fibers, as well as mechanical interactions, may affect the fiber suspension flow and the spatial distribution and orientation distribution of the fibers. When the motion is referred to axis which rotates steadily with the bulk of the fluid, the Coriolis force and centrifugal force must be supposed to act on the fluid. The centrifugal force is equivalent in its effect to a contribution to the pressure. The coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure. The fiber orientation is an important physical quantity and do not only refer to rheology of fiber suspensions. Hinze [11] derived an expression for correlation between pressure fluctuations and velocity fluctuations in turbulent motion. Anderson discussed some observation of fiber suspensions in turbulent motion. Batchelor [7] obtained the equations of motion of fiber suspensions in the flow. Kishore and Sarker [15] discussed the rate of change of vorticity covariance of MHD turbulence in a rotating system. Olson and Kerekes [18] obtained the translational and rotational dispersion coefficients on the assumption that the relative velocity between the particle and fluid could be neglected. Zhang and Lin [32] studied on the motion of particles in the turbulent pipe flow of fiber suspensions. Lin et al [16] derived the new equation of turbulent fiber suspensions and its solution and application to the pipe flow. The main aim of this study is to derive an expression for turbulent fiber motion in a rotating system with pressure-velocity correlation.

### 5.2. Mathematical model of the problem

The equation of motion and continuity for fiber suspensions in turbulent flow of viscous incompressible fluid are [16]

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j \partial x_j}+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \qquad (5.2.1)$$

$$\frac{\partial u_i}{\partial x_i}=0 \qquad (5.2.2)$$

In a rotating system, the equation of motion (5.2.1) becomes

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j \partial x_j}-2(\Omega_i u_i \eta_i)\sin\theta$$

$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \qquad (5.2.3)$$

where, $u_i$, the fluid velocity of the particle

$p=\frac{p}{\rho}+\frac{1}{2}\left|\overline{\Omega}\times\overline{u}\right|^2$ stands for the generalized pressure inclusive of potential centrifugal force,

$-2(\Omega_i u_i \eta_i)\sin\theta=-2\left(\overline{\Omega}\times\overline{u}\right)$ is the Coriolis force in which $\Omega_i$ is the angular velocity,

$\upsilon$,the kinematical viscosity of the suspending fluid;

$\eta$ is the unit vector perpendicular to $\overline{\Omega}$ and $\overline{u}$,

$\theta$ is the angle between $\overline{\Omega}$ and $\overline{u}$;

$\mu_f$, the apparent viscosity of the suspensions

$\rho$, the density of the fluid particle

$\varepsilon_{lm} = \frac{1}{2}\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right)$ is the tensor of strain rate

$I_{ij}$, the turbulent intensity of suspensions

$a_{lm}$ and $a_{ijlm}$ are the second and fourth-orientation tensors of the fiber respectively

and t is the time.

We assume that the mean velocity $\overline{U_i}$ is constant throughout the region considered and independent of time and we put

$$\left(U_i = \overline{U}_i + u_i\right)_A,$$
$$\left(U_j = \overline{U}_j + u_j\right)_B$$

The value of each term can be taken by using the equations of motion for $u_j$ at the point $B$ and for $u_i$ at the point $A$.

The equation of motion for $u_i$ at the point $A$, following equation (5.2.3) takes the form

$$\frac{\partial u_i}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon\frac{\partial^2 u_i}{\partial x_k \partial x_k} - 2(\Omega_i u_i \eta_i)\sin\theta$$
$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right] \qquad (5.2.4)$$

For an incompressible fluid $\left(u_i \frac{\partial u_k}{\partial x_k}\right)_A = 0$,

so that we can add this term to the equation (5.2.4) and thus equation (5.2.4) becomes

$$\frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A + \left(u_i\frac{\partial u_k}{\partial x_k}\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A$$

$$-2(\Omega_i u_i \eta_i)_A \sin\theta + \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right]_A \qquad (5.2.5)$$

Multiply equation (5.2.5) by $(u_j)_B$, we obtain

$$(u_j)_B\frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A(u_j)_B + (u_i)_A\left(\frac{\partial}{\partial x_k}\right)_A (u_k)_A(u_j)_B$$

$$= -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A(u_j)_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A(u_j)_B - 2(\Omega_i u_i \eta_i)_A (u_j)_B \sin\theta$$

$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right]_A (u_j)_B \qquad (5.2.6)$$

where $(u_j)_B$ is constant in a differential process at the point $A$.

Similarly, the equation of motion for $u_j$ at the point $B$, becomes

$$\frac{\partial u_j}{\partial t} + (\overline{U}_k + u_k)\frac{\partial u_j}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + \upsilon\frac{\partial^2 u_j}{\partial x_k \partial x_k} - 2(\Omega_j u_j \eta_j)\sin\theta$$

$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk}a_{lm})\varepsilon_{lm}\right] \qquad (5.2.7)$$

Since, for an incompressible fluid $\left(u_j\frac{\partial u_k}{\partial x_k}\right)_B = 0$ then the equation (5.2.7) can be written as

$$\frac{\partial}{\partial t}(u_j)_B + \left[\overline{U}_k + (u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B (u_j)_B + \left(u_j\frac{\partial u_k}{\partial x_k}\right)_B$$

$$= -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B (u_j)_B - 2(\Omega_j u_j \eta_j)_B \sin\theta$$

$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B\left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk}a_{lm})\varepsilon_{lm}\right]_B \qquad (5.2.8)$$

Multiply equation (5.2.8) by $(u_i)_A$, we get

$$(u_i)_A \frac{\partial}{\partial t}(u_j)_B + \left[\overline{U}_k + (u_k)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B (u_j)_B (u_i)_A + (u_j)_B \left(\frac{\partial}{\partial x_k}\right)_B (u_k)_B (u_i)_A$$

$$= -\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B (u_i)_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B (u_j)_B (u_i)_A - 2(\Omega_j u_j \eta_j)_B (u_i)_A \sin\theta$$

$$+ \frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B \left[a_{jklm}\varepsilon_{lm} - \frac{1}{3}(I_{jk} a_{lm})\varepsilon_{lm}\right]_B (u_i)_A \qquad (5.2.9)$$

where $(u_i)_A$ can be treated as a constant in a differential process at the point $B$.

Addition of the equations (5.2.6) and (5.2.9) gives the result

$$\frac{\partial}{\partial t}(u_i)_A(u_j)_B + \left[\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A (u_k)_A (u_j)_B + \left(\frac{\partial}{\partial x_k}\right)_B (u_i)_A (u_k)_B (u_j)_B\right] + \overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A (u_j)_B\right]$$

$$+ \overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_B (u_i)_A (u_j)_B\right] = -\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A (u_j)_B + \left(\frac{\partial}{\partial x_j}\right)_B p_B (u_i)_A\right]$$

$$-2\left[(\Omega_i u_i \eta_i)_A (u_j)_B + (\Omega_j u_j \eta_j)_B (u_i)_A\right]\sin\theta + \upsilon\left[\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A + \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\right](u_i)_A (u_j)_B$$

$$+ \frac{\mu_f}{\rho}\left[\left(\frac{\partial}{\partial x_k}\right)_A \left(a_{iklm}\varepsilon_{lm} - \frac{1}{3} I_{ik} a_{lm}\varepsilon_{lm}\right)_A (u_j)_B + \left(\frac{\partial}{\partial x_k}\right)_B \left(a_{jklm}\varepsilon_{lm} - \frac{1}{3} I_{jk} a_{lm}\varepsilon_{lm}\right)_B (u_i)_A\right] \qquad (5.2.10)$$

To find the relation of turbulent fiber motions in a rotating frame at the point $B$ to those at point $A$, it will give no difference if we take one point as the origin of $A$ or $B$ of the coordinate system.

Let us consider the point $A$ as the origin and can write

$$\zeta_k = (x_k)_B - (x_k)_A$$

Then we obtain, $\left(\frac{\partial}{\partial x_k}\right)_A = -\frac{\partial}{\partial \zeta_k}, \left(\frac{\partial}{\partial x_k}\right)_B = \frac{\partial}{\partial \zeta_k}$,

$$\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A = \left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B = \frac{\partial^2}{\partial \zeta_k \partial \zeta_k}$$

Using the above relations in equation (5.2.10) and taking ensemble average on both sides equation (5.2.10) reduces to

$$\frac{\partial}{\partial t}\overline{(u_i)_A(u_j)_B} - \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A(u_k)_A(u_j)_B} + \frac{\partial}{\partial \zeta_k}\overline{(u_i)_A(u_k)_B(u_j)_B} = -\frac{1}{\rho}\left[-\frac{\partial}{\partial \zeta_i}\overline{p_A(u_j)_B} + \frac{\partial}{\partial \zeta_j}\overline{p_B(u_i)_A}\right]$$
$$+ 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}\overline{(u_i)_A(u_j)_B} - \frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\overline{(a_{iklm}\varepsilon_{lm})_A(u_j)_B} - \frac{1}{3}\overline{(I_{ik}a_{lm}\varepsilon_{lm})_A(u_j)_B}\right]$$
$$-2\left[\overline{(\Omega_i u_i \eta_i)_A(u_j)_B} + \overline{(\Omega_j u_j \eta_j)_B(u_i)_A}\right]\sin\theta + \frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\overline{(a_{jklm}\varepsilon_{lm})_B(u_i)_A} - \frac{1}{3}\overline{(I_{jk}a_{lm}\varepsilon_{lm})_B(u_i)_A}\right] \quad (5.2.11)$$

This equation represents the mean motion equation of turbulent fiber suspensions in a rotating system with pressure-velocity correlation.

It is noted that the coefficient of $\overline{U}_k$ has been vanished. The equation (5.2.11) describes the turbulent fiber motions, where the motions with respect to a coordinate system moving with the mean velocity $\overline{U}_k$.

Equation (5.2.11) contains the double velocity correlation $\overline{(u_i)_A(u_j)_B}$, double correlations such as $\overline{p_A(u_j)_B}$, triple correlations such as $\overline{(u_i)_A(u_k)_A(u_j)_B}$ etc. where all the terms apart from one another. The correlations $\overline{p_A(u_j)_B}$ and $\overline{p_B(u_i)_A}$ form the tensors of the first order, because pressure is a scalar quantity and the triple correlations $\overline{(u_i)_A(u_k)_A(u_j)_B}$ and $\overline{(u_i)_A(u_k)_B(u_j)_B}$ form the tensors of third order. We designate the first order correlations by $(k_{p,j})_{A,B}$ , second order correlations by $(Q_{i,j})_{A,B}$ and third order correlations by $(s_{ik,j})_{A,B}$.

Therefore, we set $\left(k_{i,p}\right)_{A,B} = \overline{\left(u_i\right)_A p_B}, \left(k_{p,j}\right)_{A,B} = \overline{p_A\left(u_j\right)_B},$

$$\left(Q_{i,j}\right)_{A,B} = \overline{\left(u_i\right)_A\left(u_j\right)_B},$$

$$\left(s_{ik,j}\right)_{A,B} = \overline{\left(u_i\right)_A\left(u_k\right)_A\left(u_j\right)_B}, \left(s_{i,kj}\right)_{A,B} = \overline{\left(u_i\right)_A\left(u_k\right)_B\left(u_j\right)_B},$$

where the index $p$ indicates the pressure and is not a dummy index like $i$ or $j$ so that the summation convention does not apply to $p$.

Also the term $\overline{\left(\Omega_i u_i \eta_i\right)_A\left(u_j\right)_B}$ and $\overline{\left(\Omega_j u_j \eta_j\right)_B\left(u_i\right)_A}$ form the tensors of second order, we shall designate these by $M_{i,j}$ and $N_{i,j}$ respectively; $\overline{\left(a_{jklm}\varepsilon_{lm}\right)_B\left(u_i\right)_A}$ and $\overline{\left(I_{jk}a_{lm}\varepsilon_{lm}\right)_B\left(u_i\right)_A}$ form third order correlation tensor, we designate these by $D_{i,jk}$ and $H_{i,jk}$ respectively.

Thus we set, $\left(M_{i,j}\right)_{A,B} = \overline{\left(\Omega_i u_i \eta_i\right)_A\left(u_j\right)_B},$

$$\left(N_{i,j}\right)_{A,B} = \overline{\left(\Omega_j u_j \eta_j\right)_B\left(u_i\right)_A}$$

$$\left(D_{i,jk}\right)_{A,B} = \overline{\left(u_i\right)_A\left(a_{jklm}\varepsilon_{lm}\right)_B},$$

$$\left(D_{ik,j}\right)_{A,B} = \overline{\left(a_{iklm}\varepsilon_{lm}\right)_A\left(u_j\right)_B}$$

$$\left(H_{i,jk}\right)_{A,B} = \overline{\left(u_i\right)_A\left(I_{jk}a_{lm}\varepsilon_{lm}\right)_B},$$

$$\left(H_{ik,j}\right)_{A,B} = \overline{\left(I_{ik}a_{lm}\varepsilon_{lm}\right)_A\left(u_j\right)_B}$$

If we use the above relations of first, second and third order correlations in equation (5.2.11) then we obtain

$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}S_{ik,j} + \frac{\partial}{\partial \zeta_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j} + \frac{\partial}{\partial \zeta_j}K_{i,p}\right) + 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$

$$-2\left(M_{i,j} + N_{i,j}\right)\sin\theta + \frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial \zeta_k}\left(D_{ik,j} - \frac{1}{3}H_{ik,j}\right) + \frac{\partial}{\partial \zeta_k}\left(D_{i,jk} - \frac{1}{3}H_{i,jk}\right)\right]$$

$$\text{or,} \frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}S_{ik,j} + \frac{\partial}{\partial \zeta_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j} + \frac{\partial}{\partial \zeta_j}K_{i,p}\right) + 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$

$$-2(M_{i,j} + N_{i,j})\sin\theta + \frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[(D_{i,jk} - D_{ik,j}) + \frac{1}{3}(H_{ik,j} - H_{i,jk})\right] \qquad (5.2.12)$$

where all correlations refer to the two points $A$ and $B$.

Now for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero, that is $(k_{p,j})_{A,B} = 0, (k_{i,p})_{A,B} = 0$

For an isotropic turbulence the condition of invariance under reflection with respect to point $A$,

$$\overline{(u_i)_A (u_k)_B (u_j)_B} = -\overline{(u_k)_A (u_j)_A (u_i)_B}$$

or, $$(s_{i,kj})_{A,B} = -(s_{kj,i})_{A,B}$$

so that $$(D_{i,jk})_{A,B} = -(D_{jk,i})_{A,B},$$

$$(H_{i,jk})_{A,B} = -(H_{jk,i})_{A,B}$$

Thus equation (5.2.12) can be written as

$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}(S_{ik,j} + S_{kj,i}) = 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j} - 2(M_{i,j} + N_{i,j})\sin\theta$$

$$+\frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial \zeta_k}(D_{jk,i} + D_{ik,j}) + \frac{1}{3}\frac{\partial}{\partial \zeta_k}(H_{ik,j} + H_{jk,i})\right] \qquad (5.2.13)$$

The term $\frac{\partial}{\partial \zeta_k}(S_{ik,j} + S_{kj,i})$, $\frac{\partial}{\partial \zeta_k}(D_{jk,i} + D_{ik,j})$ and $\frac{\partial}{\partial \zeta_k}(H_{ik,j} + H_{jk,i})$ form the tensors of second order, we designate these by $S_{i,j}, D_{i,j}$ and $H_{i,j}$ respectively, that is

$$S_{i,j} = \frac{\partial}{\partial \zeta_k}(S_{ik,j} + S_{kj,i}),$$

$$D_{i,j} = \frac{\partial}{\partial \zeta_k}(D_{jk,i} + D_{ik,j}),$$

and $H_{i,j} = \dfrac{\partial}{\partial \zeta_k}\left(H_{ik,j} + H_{jk,i}\right).$

Also, the term $\left(M_{i,j} + N_{i,j}\right)$ form the second order tensor, say $W_{i,j}$

that is

$$W_{i,j} = \left(M_{i,j} + N_{i,j}\right).$$

Therefore equation (5.2.13) gives the result

$$\frac{\partial}{\partial t} Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} - 2W_{i,j} \sin\theta - \frac{\mu_f}{\rho}\left(D_{i,j} - \frac{1}{3} H_{i,j}\right) \qquad (5.2.14)$$

This is the resulting equation of the turbulent fiber motion in a rotating system in terms of the correlation tensors of second order.

For non-rotating system, $W_{i,j} = 0$ so that equation (5.2.14) takes the form

$$\frac{\partial}{\partial t} Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} - \frac{\mu_f}{\rho}\left(D_{i,j} - \frac{1}{3} H_{i,j}\right) \qquad (5.2.15)$$

The equation (5.2.15) describes the turbulent fiber motion for non-rotating system in terms of the correlation tensors of second order.

If there are no effects of fiber suspension in the flow field then $\mu_f = 0$ and hence the equation (5.2.15) takes the form

$$\frac{\partial}{\partial t} Q_{i,j} - S_{i,j} = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} \qquad (5.2.16)$$

The equation (5.2.16) represents the turbulent motion in terms of correlation tensors of second order which is the same as obtained by Hinze [11].

## 5.3. Discussion and conclusion

Fiber motion in turbulent flow in a rotating system has been obtained from the equation of mean fiber motion by taking the average procedure and including the effect of coriolis force due to rotation and fiber suspensions with the correlations between the pressure fluctuations and velocity fluctuations at two points of the fluid flow. Turbulent fiber motion has been discussed here with the aid of pressure-velocity correlation in a rotating system, where the Coriolis force and centrifugal force act on the fluid. Therefore, the Coriolis force due to rotation plays an important role in a rotating system of fluid dynamics. The discussion provides the equation of mean turbulent motion of fiber suspensions due to rotation. Fiber suspensions in a turbulent flow undergo a mean motion due to the mean fluid velocity and a random motion due to the fluctuating component of fluid velocity. Fiber suspensions in a turbulent flow undergo mean motion due to the mean fluid velocity and random motion due to the fluctuating component of fluid velocity. The velocity of fiber fluctuates around the mean velocity of flow. Fluctuation velocity of turbulence at the two points $A$ and $B$ of the flow field leads to a weakening of the concentration of the fiber orientation distribution on small angle. This concentration leads to be weaker and orientation distribution of fiber becomes more uniform as Reynolds numbers increases and flow fluctuation velocity strengthens.

Because the fluctuation velocity is isotropic, the translation and rotation of fiber are also isotropic in turbulence. The distribution function of fiber $\theta$ changes in the range from $0^{\circ}$ to $90^{\circ}$. In a rotating system angular velocity plays also a vital role in the flow field. Since the fluctuation of flow velocity gradient is random and changes around zero, then the angular velocity of fiber fluctuates around zero.

If we take the mean square deviation of fiber angular velocity for three components are as

$$\Omega_x'' = \sqrt{\frac{\sum_{n=1}^{N}\left(\Omega_{nx} - \overline{\Omega}_x\right)^2}{N}},$$

$$\Omega_y'' = \sqrt{\frac{\sum_{n=1}^{N}\left(\Omega_{ny} - \overline{\Omega}_y\right)^2}{N}},$$

$$\Omega_z'' = \sqrt{\frac{\sum_{n=1}^{N}\left(\Omega_{nz} - \overline{\Omega}_z\right)^2}{N}}.$$

where, $\overline{\Omega}_x, \overline{\Omega}_y, \overline{\Omega}_z$ are the mean angular velocity in $x, y$ and $z$ direction in a circular cross section, $N$ is the number of fibers, then the mean square deviation of fiber velocity increases with the increase of $\mathrm{Re}$, which means that the fluctuation of angular velocity enhances.

For a turbulent pipe flow, the turbulent intensity of velocity gradient on flow direction is stronger than that on lateral direction. Hence, velocity gradient on the flow direction leads to the angular velocity of fiber on $x$ and $y$ direction, while velocity gradient on the lateral direction leads to that on $z$ direction. Thus the angular velocity of fiber on $x$ and $y$ direction is wider than that on $z$ direction. The resulting equation states that as Reynolds number increases and fluctuation velocity of fluid enhances, the fluctuation of flow velocity gradient strengthens, which results in a stronger rotation of fiber.

# CHAPTER SIX
# Fibre Motion in Dusty Fluid Turbulent Flow in a Rotating System

# CHAPTER SIX

## Fiber Motion in Dusty Fluid Turbulent Flow in a Rotating System

### 6.1. Introduction

The turbulent flow of fiber suspensions can be found in many areas of industry, such as the production of the composite materials, environmental engineering, chemical engineering, textile industry, paper making and so on. The fiber-fluid interaction depends heavily on the nature and magnitude of the interactions. Long range and short-range hydrodynamic interactions between fibers, as well as mechanical interactions, may affect the fiber suspension flow and the spatial distribution and orientation distribution of the fibers. The behavior of fiber suspensions in presence of dust particles in a turbulent fluid depends on the concentration of the particles and on the size of the particles with respect to the scale of turbulent fluid. The fluid must be affected when the motion is referred to axis which rotates steadily with the bulk of the fluid, the Coriolis force and centrifugal force. The Coriolis force due to rotation plays an important role in a rotating system of turbulent flow, while the centrifugal force with the potential is incorporated into the pressure. Saffman [22] observed the effect of dust particles of an incompressible flow and derived an equation that described the motion of a fluid containing small dust particles. Kishore and Sarker [15] discussed the rate of change of vorticity covariance of MHD turbulence in a rotating system. Anderson [2] discussed on some observation of fiber suspensions in turbulent motion. Batchelor [7] obtained the equations of motion of fiber suspensions in the flow. Hinze [11] obtained an expression for correlation between pressure fluctuations and velocity fluctuations in turbulent motion. Agermann and kohler [1] studied on rotational and translational dispersion of fibers in turbulent flow by assuming the dimension of fibers to be less than that of smallest eddies in the flow. Olson and Kerekes [18] obtained the translational and rotational dispersion coefficients on the assumption that the relative velocity between the particle and fluid could be neglected. Zhao et al.[33] discussed the complexity of fiber suspension results from the effects of particle. Lin et al. [17] investigated the effect of fibers on the turbulent property of flow, where the effect of the fluid on the fibers was neglected. Zhang and Lin [32] studied on the motion of particles in the turbulent pipe flow of fiber suspensions. Lin et al. [16] derived the new equation of turbulent fiber suspensions and its solution. They also verified the equations and their solutions by applying to a turbulent pipe flow. The main aim of this study is to derive an equation of fiber motion for dusty fluid turbulent flow in a rotating system with the pressure-velocity correlation.

## 6.2. Mathematical Analysis

Let us assume that the fluid is to be incompressible. Accordingly, the derivation are based on the following set of equations

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\upsilon\frac{\partial^2 u_i}{\partial x_j \partial x_j}+\frac{KN}{\rho}(v_i-u_i)-2(\Omega_i u_i \eta_i)\sin\theta$$

$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_j}\left[a_{ijlm}\varepsilon_{lm}-\frac{1}{3}\left(I_{ij}a_{lm}\right)\varepsilon_{lm}\right] \qquad (6.2.1)$$

$$\frac{\partial u_i}{\partial x_i}=0 \qquad (6.2.2)$$

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial u_i}{\partial x_j} = -\frac{K}{m_s}(u_i - v_i) \qquad (6.2.3)$$

where $u_i(x,t)$, the fluid velocity components

$v_i(x,t)$, the solid particles (dust) velocity components

$p = \frac{p}{\rho} + \frac{1}{2}\left|\overline{\Omega} \times \overline{u}\right|^2$ stands for the generalized pressure inclusive of potential centrifugal force

$\upsilon$,the kinematical viscosity of the suspending fluid

$-2(\Omega_i u_i \eta_i)\sin\theta = -2(\overline{\Omega} \times \overline{u})$ is the Coriolis force in which $\Omega_i$ is the angular velocity,

$\eta$ is the unit vector perpendicular to $\overline{\Omega}$ and $\overline{x}$,

$\theta$ is the angle between $\overline{\Omega}$ and $\overline{x}$

$m_s = \frac{4}{3}\pi R_s^3 \rho_s$, the mass of a single spherical dust particles of radius $R_s$

$\upsilon =$ constant is the molecular kinematic viscosity

$K = 6\pi R_s \rho v$, the Stoke's drag formula

$N$, the number density of dust particles;

$\frac{KN}{\rho} = f$, has dimension of frequency

$\mu_f$, the apparent viscosity of the suspensions

$\rho$, the density of the fluid particle

$\varepsilon_{lm} = \frac{1}{2}\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right)$ is the tensor of strain rate

$I_{ij}$, the turbulent intensity of fiber suspensions

$a_{lm}$ and $a_{ijlm}$ are the second and fourth-orientation tensors of the fiber respectively and $t$ is the time.

We assume that the mean velocity $\overline{U_i}$ is constant throughout the region considered and independent of time and we put

$$\left(U_i = \overline{U}_i + u_i\right)_A, \left(U_j = \overline{U}_j + u_j\right)_B$$

The value of each term can be obtained by using the equations of motion for $u_j$ at the point $B$ and for $u_i$ at the point $A$.

The equation of motion for $u_i$ at the point $A$, obtain from equation (6.2.1)

$$\frac{\partial u_i}{\partial t} + \left(\overline{U}_k + u_k\right)\frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \upsilon\frac{\partial^2 u_i}{\partial x_k \partial x_k} + f(v_i - u_i) - 2(\Omega_i u_i \eta_i)\sin\theta$$

$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right]$$

For an incompressible fluid $\left(u_i \dfrac{\partial u_k}{\partial x_k}\right)_A = 0$ so that we can add this term to the above equation and thus above equation becomes

$$\frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A + \left(u_i\frac{\partial u_k}{\partial x_k}\right)_A = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A + \upsilon\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A$$

$$+ f(v_i - u_i)_A - 2(\Omega_i u_i \eta_i)_A \sin\theta$$

$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right]_A \quad (6.2.4)$$

Multiply equation (6.2.4) by $(u_j)_B$ we obtain

$$(u_j)_B\frac{\partial}{\partial t}(u_i)_A + \left[\overline{U}_k + (u_k)_A\right]\left(\frac{\partial}{\partial x_k}\right)_A (u_i)_A(u_j)_B + (u_i)_A\left(\frac{\partial}{\partial x_k}\right)_A (u_k)_A(u_j)_B = -\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\right)_A p_A(u_j)_B$$

$$+\nu\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_A (u_i)_A(u_j)_B + f(v_i - u_i)_A(u_j)_B - 2(\Omega_i u_i \eta_i)_A(u_j)_B \sin\theta$$

$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_A\left[a_{iklm}\varepsilon_{lm} - \frac{1}{3}(I_{ik}a_{lm})\varepsilon_{lm}\right]_A (u_j)_B \quad (6.2.5)$$

where, $(u_j)_B$ can be treated as a constant in a differential process at the point $A$.

Similarly, the equation of motion for $u_j$ at the point $B$,

$$\frac{\partial u_j}{\partial t}+\left(\overline{U}_k+u_k\right)\frac{\partial u_j}{\partial x_k}=-\frac{1}{\rho}\frac{\partial p}{\partial x_j}+\nu\frac{\partial^2 u_j}{\partial x_k \partial x_k}+f\left(v_j-u_j\right)-2\left(\Omega_j u_j \eta_j\right)\sin\theta$$

$$+\frac{\mu_f}{\rho}\frac{\partial}{\partial x_k}\left[a_{jklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]$$

Since, for an incompressible fluid $\left(u_j\frac{\partial u_k}{\partial x_k}\right)_B=0$ then the equation (6.2.5) can be written as

$$\frac{\partial}{\partial t}\left(u_j\right)_B+\left[\overline{U}_k+\left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B\left(u_j\right)_B+\left(u_j\frac{\partial u_k}{\partial x_k}\right)_B=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B+\nu\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\left(u_j\right)_B$$

$$+f\left(v_j-u_j\right)_B-2\left(\Omega_j u_j \eta_j\right)_B\sin\theta+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B\left[a_{jklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_B \quad (6.2.6)$$

Multiplying equation (6.2.6) by $(u_i)_A$, we get

$$\left(u_i\right)_A\frac{\partial}{\partial t}\left(u_j\right)_B+\left[\overline{U}_k+\left(u_k\right)_B\right]\left(\frac{\partial}{\partial x_k}\right)_B\left(u_j\right)_B\left(u_i\right)_A+\left(u_j\right)_B\left(\frac{\partial}{\partial x_k}\right)_B\left(u_k\right)_B\left(u_i\right)_A=-\frac{1}{\rho}\left(\frac{\partial}{\partial x_j}\right)_B p_B\left(u_i\right)_A$$

$$+\nu\left(\frac{\partial^2}{\partial x_k \partial x_k}\right)_B\left(u_j\right)_B\left(u_i\right)_A+f\left(v_j-u_j\right)_B\left(u_i\right)_A-2\left(\Omega_j u_j \eta_j\right)_B\left(u_i\right)_A\sin\theta$$

$$+\frac{\mu_f}{\rho}\left(\frac{\partial}{\partial x_k}\right)_B\left[a_{jklm}\varepsilon_{lm}-\frac{1}{3}\left(I_{jk}a_{lm}\right)\varepsilon_{lm}\right]_B\left(u_i\right)_A \quad (6.2.7)$$

where $(u_i)_A$ can be treated as a constant in a differential process at the point $B$.

Addition of the equations (6.2.5) and (6.2.7) give the result

$$\frac{\partial}{\partial t}(u_i)_A(u_j)_B+\left[\left(\frac{\partial}{\partial x_k}\right)_A(u_i)_A(u_k)_A(u_j)_B+\left(\frac{\partial}{\partial x_k}\right)_B(u_i)_A(u_k)_B(u_j)_B\right]+\overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_A(u_i)_A(u_j)_B\right]$$

$$+\overline{U}_k\left[\left(\frac{\partial}{\partial x_k}\right)_B(u_i)_A(u_j)_B\right]=-\frac{1}{\rho}\left[\left(\frac{\partial}{\partial x_i}\right)_A p_A(u_j)_B+\left(\frac{\partial}{\partial x_j}\right)_B p_B(u_i)_A\right]$$

$$+\upsilon\left[\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A+\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B\right](u_i)_A(u_j)_B$$

$$+f\left[(v_i-u_i)_A(u_j)_B+(v_j-u_j)_B(u_i)_A\right]-2\left[(\Omega_i u_i\eta_i)_A(u_j)_B+(\Omega_j u_j\eta_j)_B(u_i)_A\right]\sin\theta$$

$$+\frac{\mu_f}{\rho}\left[\left(\frac{\partial}{\partial x_k}\right)_A\left(a_{iklm}\varepsilon_{lm}-\frac{1}{3}I_{ik}a_{lm}\varepsilon_{lm}\right)_A(u_j)_B+\left(\frac{\partial}{\partial x_k}\right)_B\left(a_{jklm}\varepsilon_{lm}-\frac{1}{3}I_{jk}a_{lm}\varepsilon_{lm}\right)_B(u_i)_A\right] \quad (6.2.8)$$

To find the relation of turbulent fiber motions in presence of dust particles in a rotating system at the point $B$ to those at point $A$, it will give no difference if we take one point as the origin of $A$ or $B$ of the coordinate system. Let us consider the point $A$ as the origin and can write

$$\zeta_k=(x_k)_B-(x_k)_A$$

Then we obtain, $$\left(\frac{\partial}{\partial x_k}\right)_A=-\frac{\partial}{\partial\zeta_k},\left(\frac{\partial}{\partial x_k}\right)_B=\frac{\partial}{\partial\zeta_k}$$

$$\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_A=\left(\frac{\partial^2}{\partial x_k\partial x_k}\right)_B=\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}$$

Using the above relations in equation (6.2.8) and taking ensemble average on both sides, then equation (6.2.8) becomes

$$\frac{\partial}{\partial t}\overline{(u_i)_A(u_j)_B}-\frac{\partial}{\partial\zeta_k}\overline{(u_i)_A(u_k)_A(u_j)_B}+\frac{\partial}{\partial\zeta_k}\overline{(u_i)_A(u_k)_B(u_j)_B}=-\frac{1}{\rho}\left[-\frac{\partial}{\partial\zeta_i}\overline{p_A(u_j)_B}+\frac{\partial}{\partial\zeta_j}\overline{p_B(u_i)_A}\right]$$

$$+2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}\overline{(u_i)_A(u_j)_B}+f\left[\overline{(v_i)_A(u_j)_B}-2\overline{(u_i)_A(u_j)_B}+\overline{(u_i)_A(v_j)_B}\right]$$

$$-2\left[\overline{(\Omega_i u_i\eta_i)_A(u_j)_B}+\overline{(\Omega_j u_j\eta_j)_B(u_i)_A}\right]\sin\theta \quad (6.2.9)$$

This equation represents the equation of mean motion of turbulent fiber suspensions in a rotating system under the influence of dust particles with pressure-velocity correlation.

It is noted that the coefficient of $\overline{U}_k$ has been vanished. The equation (6.2.9) describes the turbulent fiber motion in a rotating system in presence of dust particles, where the motions with respect to a coordinate system moving with the mean velocity $\overline{U}_k$.

Equation (6.2.9) contains the double velocity correlation $\overline{(u_i)_A(u_j)_B}$, double velocity correlation between dust particles and the fluid such as $\overline{(v_i)_A(u_j)_B}$, double correlations such as $\overline{p_A(u_j)_B}$, triple correlations such as $\overline{(u_i)_A(u_k)_A(u_j)_B}$ where all the terms apart from one to another. The correlations $\overline{p_A(u_j)_B}$ and $\overline{p_B(u_i)_A}$ form the tensors of the first order, because pressure is a scalar quantity and the triple correlations $\overline{(u_i)_A(u_k)_A(u_j)_B}$ and $\overline{(u_i)_A(u_k)_B(u_j)_B}$ form the tensors of third order.

We shall designate the first order correlations by $(k_{p,j})_{A,B}$ , second order correlations by $(Q_{i,j})_{A,B}$ and third order correlations by $(s_{ik,j})_{A,B}$.

Therefore, we set $(k_{i,p})_{A,B} = \overline{(u_i)_A p_B}, (k_{p,j})_{A,B} = \overline{p_A(u_j)_B}$,

$$(s_{ik,j})_{A,B} = \overline{(u_i)_A(u_k)_A(u_j)_B},$$

$$(s_{i,kj})_{A,B} = \overline{(u_i)_A(u_k)_B(u_j)_B},\ (Q_{i,j})_{A,B} = \overline{(u_i)_A(u_j)_B}$$

$$(F_{i,j})_{A,B} = \overline{(v_i)_A(u_j)_B}, (G_{i,j})_{A,B} = \overline{(u_i)_A(v_j)_B}$$

where the index $p$ indicates the pressure and is not a dummy index like $i$ or $j$ so that the summation convention does not apply to $p$.

Also the term $\overline{(\Omega_i u_i \eta_i)_A (u_j)_B}$ and $\overline{(\Omega_j u_j \eta_j)_B (u_i)_A}$ form the tensors of second order, we shall designate these by $M_{i,j}$ and $N_{i,j}$ respectively; $\overline{(a_{jklm}\varepsilon_{lm})_B (u_i)_A}$ and $\overline{(I_{jk} a_{lm}\varepsilon_{lm})_B (u_i)_A}$ form third order correlations, we designate these by $D_{i,jk}$ and $H_{i,jk}$ respectively.

Thus we set
$$(M_{i,j})_{A,B} = \overline{(\Omega_i u_i \eta_i)_A (u_j)_B},$$

$$(N_{i,j})_{A,B} = \overline{(\Omega_j u_j \eta_j)_B (u_i)_A}$$

$$(D_{i,jk})_{A,B} = \overline{(u_i)_A (a_{jklm}\varepsilon_{lm})_B},$$

$$(D_{ik,j})_{A,B} = \overline{(a_{iklm}\varepsilon_{lm})_A (u_j)_B},$$

$$(H_{i,jk})_{A,B} = \overline{(u_i)_A (I_{jk} a_{lm}\varepsilon_{lm})_B},$$

$$(H_{ik,j})_{A,B} = \overline{(I_{ik} a_{lm}\varepsilon_{lm})_A (u_j)_B}.$$

If we use the above relations of first, second and third order correlations in equation (6.2.9) then we obtain

$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}S_{ik,j} + \frac{\partial}{\partial \zeta_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j} + \frac{\partial}{\partial \zeta_j}K_{i,p}\right) + 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$
$$+ f\left(F_{i,j} - 2Q_{i,j} + G_{i,j}\right) - 2\left(M_{i,j} + N_{i,j}\right)\sin\theta$$
$$+ \frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial \zeta_k}\left(D_{ik,j} - \frac{1}{3}H_{ik,j}\right) + \frac{\partial}{\partial \zeta_k}\left(D_{i,jk} - \frac{1}{3}H_{i,jk}\right)\right]$$

or
$$\frac{\partial}{\partial t}Q_{i,j} - \frac{\partial}{\partial \zeta_k}S_{ik,j} + \frac{\partial}{\partial \zeta_k}S_{i,kj} = -\frac{1}{\rho}\left(-\frac{\partial}{\partial \zeta_i}K_{p,j} + \frac{\partial}{\partial \zeta_j}K_{i,p}\right) + 2\upsilon\frac{\partial^2}{\partial \zeta_k \partial \zeta_k}Q_{i,j}$$
$$+ f\left(F_{i,j} - 2Q_{i,j} + G_{i,j}\right) - 2\left(M_{i,j} + N_{i,j}\right)\sin\theta$$
$$+ \frac{\mu_f}{\rho}\frac{\partial}{\partial \zeta_k}\left[\left(D_{i,jk} - D_{ik,j}\right) + \frac{1}{3}\left(H_{ik,j} - H_{i,jk}\right)\right] \qquad (6.2.10)$$

where all correlations refer to the two points $A$ and $B$.

Now for an isotropic turbulence of an incompressible flow, the double pressure-velocity correlations are zero, that is

$$(k_{p,j})_{A,B} = 0,$$
$$(k_{i,p})_{A,B} = 0.$$

In an isotropic turbulence it follows from the condition of invariance under reflection with respect to point $A$

$$\overline{(u_i)_A (u_k)_B (u_j)_B} = -\overline{(u_k)_A (u_j)_A (u_i)_B}$$

or $$(s_{i,kj})_{A,B} = -(s_{kj,i})_{A,B},$$

so that $$(D_{i,jk})_{A,B} = -(D_{jk,i})_{A,B},$$

$$(H_{i,jk})_{A,B} = -(H_{jk,i})_{A,B}.$$

Thus equation (6.2.10) reduces to

$$\frac{\partial}{\partial t} Q_{i,j} - \frac{\partial}{\partial \zeta_k}(S_{ik,j} + S_{kj,i}) = 2\upsilon \frac{\partial^2}{\partial \zeta_k \partial \zeta_k} Q_{i,j} + f(F_{i,j} - 2Q_{i,j} + G_{i,j})$$

$$-2(M_{i,j} + N_{i,j})\sin\theta + \frac{\mu_f}{\rho}\left[-\frac{\partial}{\partial \zeta_k}(D_{jk,i} + D_{ik,j}) + \frac{1}{3}\frac{\partial}{\partial \zeta_k}(H_{ik,j} + H_{jk,i})\right] \quad (6.2.11)$$

The term $\frac{\partial}{\partial \zeta_k}(S_{ik,j} + S_{kj,i})$, $\frac{\partial}{\partial \zeta_k}(D_{jk,i} + D_{ik,j})$ and $\frac{\partial}{\partial \zeta_k}(H_{ik,j} + H_{jk,i})$ form the tensors of second order, we shall designate these by $S_{i,j}, D_{i,j}$ and $H_{i,j}$ respectively, that is

$$S_{i,j} = \frac{\partial}{\partial \zeta_k}(S_{ik,j} + S_{kj,i}),$$

$$D_{i,j} = \frac{\partial}{\partial \zeta_k}(D_{jk,i} + D_{ik,j})$$

and $$H_{i,j} = \frac{\partial}{\partial \zeta_k}(H_{ik,j} + H_{jk,i})$$

Also, the term $\left(M_{i,j}+N_{i,j}\right)$ form the second order tensor, say $W_{i,j}$ so that

$$W_{i,j}=\left(M_{i,j}+N_{i,j}\right)$$

Therefore equation (6.2.11) gives the result

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}+f\left(F_{i,j}-2Q_{i,j}+G_{i,j}\right)$$

$$-2W_{i,j}\sin\theta-\frac{\mu_f}{\rho}\left(D_{i,j}-\frac{1}{3}H_{i,j}\right)\qquad(6.2.12)$$

This is the resulting equation of the turbulent fiber motion in a rotating system under the influence of dust particles in terms of the correlation tensors of second order.

In absence of dust particles, $f=0$ then equation (6.2.12) reduces to

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}-2W_{i,j}\sin\theta-\frac{\mu_f}{\rho}\left(D_{i,j}-\frac{1}{3}H_{i,j}\right)\qquad(6.2.13)$$

This equation describes the turbulent fiber motion in a rotating system in terms of the correlation tensors of second order which is the same as obtained earlier.

For non-rotating system, $W_{i,j}=0$ so that equation (6.2.13) takes the form

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}-\frac{\mu_f}{\rho}\left(D_{i,j}-\frac{1}{3}H_{i,j}\right)\qquad(6.2.14)$$

This equation describes the turbulent fiber motion for non-rotating system in terms of the correlation tensors of second order which is the same as obtained earlier.

If there are no effects of fiber suspension in the flow field then $\mu_f=0$ so that the equation (6.2.14) gives

$$\frac{\partial}{\partial t}Q_{i,j}-S_{i,j}=2\upsilon\frac{\partial^2}{\partial\zeta_k\partial\zeta_k}Q_{i,j}\qquad(6.2.15)$$

This equation represents the turbulent motion in terms of correlation tensors of second order which is the same as obtained by Hinze [11].

### 6.3. Discussion and Conclusion:

The equation of fiber motion for dusty fluid turbulent flow in a rotating system has been derived by taking average procedure, which includes the effect of dust particles, Coriolis force due to rotation and pressure-velocity fluctuations at two points of the fluid flow. The discussion provides the equation of fiber mean motion in a rotating system as well as for the resulting dusty turbulent fiber motion. The occurrence of the turbulent flow will depend on the values of the non-dimensional number known as critical Reynolds number, which varies from 2000 to 2300. The flow will be turbulent if the Reynolds number $(R_e)$ is greater than the critical Reynolds number $(R_{cr})$, so that the turbulent flow occurs at high Reynolds number. If the Reynolds number increases from 1600 to 2500 then the flow converts to turbulent flow from laminar flow, the orientation distribution of fiber changes in a range. It is clear that turbulence has effect on the orientation distribution of fiber and dust particles in a rotating system. Since, the fluctuation velocity is isotropic, the translation and rotation of fiber are also isotropic in turbulence. The distribution function of fiber $\theta$ changes in the range from $0^\circ \text{to} 90^\circ$. In a rotating system angular velocity plays also a vital role in the flow field. Since the fluctuation of flow velocity gradient is random and changes around zero, then the angular velocity of fiber fluctuates around zero. If we take the mean square deviation of fiber angular velocity for three components are as

$$\Omega_x'' = \sqrt{\frac{\sum_{n=1}^{N}\left(\Omega_{nx} - \overline{\Omega}_x\right)^2}{N}},$$

$$\Omega_y'' = \sqrt{\frac{\sum_{n=1}^{N}\left(\Omega_{ny} - \overline{\Omega}_y\right)^2}{N}},$$

$$\Omega_z'' = \sqrt{\frac{\sum_{n=1}^{N}\left(\Omega_{nz} - \overline{\Omega}_z\right)^2}{N}}.$$

where, $\overline{\Omega}_x, \overline{\Omega}_y, \overline{\Omega}_z$ are the mean angular velocity in $x, y$ and $z$ direction in a circular cross section, $N$ is the number of fibers, then the mean square deviation of fiber velocity increases with the increase of $R_e$, which means that the fluctuation of angular velocity enhances.

For a turbulent pipe flow, the turbulent intensity of velocity gradient on flow direction is stronger than that on lateral direction. Hence, velocity gradient on the flow direction leads to the angular velocity of fiber on $x$ and $y$ direction, while velocity gradient on the lateral direction leads to that on $z$ direction. Thus the angular velocity of fiber on $x$ and $y$ direction is wider than that on $z$ direction.

Fibers suspensions in a turbulent fluid undergo a mean motion due to the mean fluid velocity and a random motion due to the fluctuating component of fluid velocity. Fiber suspensions in a turbulent fluid undergo mean motion due to the mean fluid velocity and random motion due to the fluctuating component of fluid velocity. The velocity of fiber fluctuates around the mean velocity of flow. Fluctuation velocity of turbulence at the two points $A$ and $B$ of the flow field leads to a weakening of the concentration of the fiber orientation distribution on small angle. In presence of dust particles this concentration leads to be weaker and orientation distribution of fiber becomes more uniform as Reynolds numbers increases.

In a rotating system, Coriolis force and centrifugal force act on the fluid. For a non-rotating system, the velocity of fiber has the same fluctuation property as fluid velocity due to the strong following ability of fiber. The fluctuation velocity of fiber on flow direction is more energetic than that on lateral direction. Thus the resulting equation demonstrates that in presence of dust particles the fluctuation velocity gradient strengths with the increases of Reynolds number due to rotation of fiber.

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